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SURFACE GALVANOMAGNETIC PHENOMENA IN THE CDW PHASE OF WEYL SEMIMETALS

(Bachelor's thesis)

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Abstract

We study surface galvanomagnetic phenomena in the Charge Density Wave (CDW) phase of Weyl semimetals, where the bulk of the system becomes insulating. The surface of the system remains conducting. In the presence of a magnetic field in the CDW phase of Weyl semimetals occurs the periodic cyclotron motion of the Weyl fermions. In the limit of a small value of the CDW gap in the leading order, we obtain the formula for the surface state electron dispersion. It allows finding the period of cyclotron motion of Weyl electrons using a semiclassical approach. We find that the period is dominated by the regions of momentum space, where the different Fermi arcs interconnect due to the electron scattering off the CDW order parameter. Our theory predicts that in these regions the traversal time is long (inversely proportional to the CDW gap).

Contents

Introduction

The Dirac equation [1] in three spatial dimensions for the wave function of the relativistic particle with the spin $\boldsymbol{s} = \frac{1}{2}$ ($\hbar = 1$)

$$i\frac{\partial \boldsymbol{\Psi}}{\partial t} = \widehat{H}\boldsymbol{\Psi}. \tag{0.1}$$

The corresponding Hamiltonian in this equation

$$\widehat{H} = c\boldsymbol{p}\boldsymbol{\alpha} + mc^2\widehat{\alpha}_0, \tag{0.2}$$

where the c is the speed of ligth, m and $\boldsymbol{p}$ are the rest mass and momentum of the particle respectively. The matrices $\widehat{\alpha}_j$ are Hermitian and they all satisfy the relations

$$\begin{gathered} \widehat{\alpha}_j^2 = \mathbb{1}_4, \\ \widehat{\alpha}_i\widehat{\alpha}_j + \widehat{\alpha}_j\widehat{\alpha}_i = 0, \quad (i \neq j). \end{gathered} \tag{0.3}$$

In three spatial dimensions the corresponding matrices can be represented as

$$\begin{aligned} &\widehat{\alpha}_i = \widehat{\sigma}_x \otimes \widehat{\sigma}_i, \quad (i \neq 0) \\ &\widehat{\alpha}_0 = \widehat{\sigma}_z \otimes \mathbb{1}_2. \end{aligned} \tag{0.4}$$

In 1929, Hermann Weyl noticed [2] that in the case of massless particles the Dirac equation can be simplified

$$i\frac{\partial \boldsymbol{\psi}_\pm}{\partial t} = \pm c\boldsymbol{p}\boldsymbol{\sigma}\boldsymbol{\psi}_\pm, \tag{0.5}$$

where $\boldsymbol{\psi}_\pm = \boldsymbol{\Psi}_1 \pm \boldsymbol{\Psi}_2$, $\boldsymbol{\Psi} = (\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2)^T$.

In condensed matter physics we should replace the speed of light with the Fermi velocity. Therefore, the resulting dispersion

$$E(p) = \pm v_F p, \quad p = \sqrt{p_x^2 + p_y^2 + p_z^2}, \tag{0.6}$$

where "$\pm$" corresponds to a right (left) moving particle, which is termed chiral or Weyl fermions. Points of band touching are called Weyl nodes. Thus, the Weyl semimetal is a crystalline conductor with the linear dispersion law near the Weyl nodes and with energy excitations that are relativistic massless fermions. Besides, it's important that any small perturbation can't open a gap, it just shifts the position of the Weyl node.

Furthermore, Weyl nodes are monopoles of Berry flux. It can be shown that in Weyl semimetals because of the vanishing of Berry flux through the entire Brillouin zone Weyl nodes come only in pairs of opposite chirality. This is a corollary of the theorem of Nielsen and Ninomiya [3]. Besides, Weyl semimetal can't exist without broken time-reversal (TR) or inversion (I) symmetry. In the presence of TR symmetry the following is true $\boldsymbol{k} \to -\boldsymbol{k}, \boldsymbol{\sigma} \to -\boldsymbol{\sigma}$ that is why this allows for the minimal number of the Weyl nodes is equal four.

Let's consider now the Fermi arc surface states in Weyl semimetal in the situation when the bulk is gapless. In this case, the electron dynamics may be described by the Hamiltonian

$$\widehat{H} = v_F \boldsymbol{p\sigma}. \tag{0.7}$$

The surface state electron dispersion may be obtained from the corresponding equation and boundary conditions that correspond to a vanishing current normal to the boundary

$$(v_p - e^{i\chi}u_p)\mid_{z=0} = 0. \tag{0.8}$$

In the result we get

$$\varepsilon = vp_x \cos\chi + vp_y \sin\chi. \tag{0.9}$$

The plot of both surface and bulk bands is shown in Fig. 1

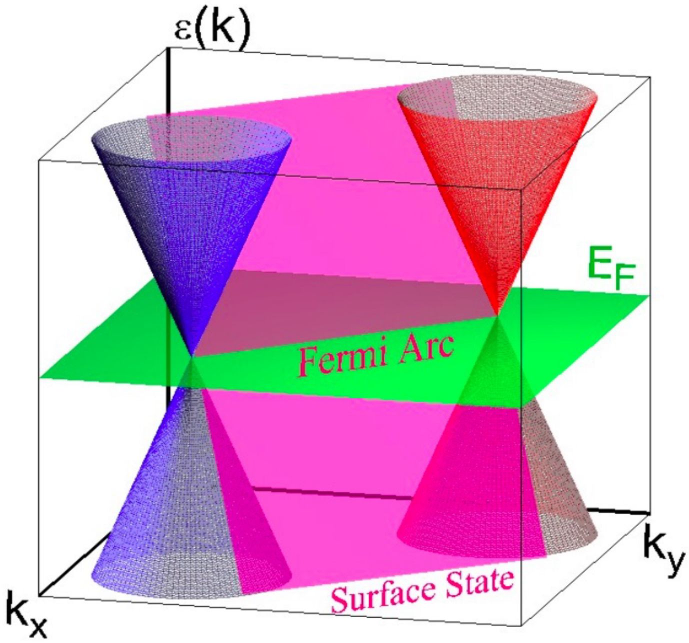

Figure 1: Adopted from work [4]. Surface bands (pink) and Weyl's cones (red and blue). There is also a Fermi level (green).

As discussed above, to open a gap in dispersion the perturbation must have certain properties. It has been shown [5] that if interaction hybridizes fermions with opposite chirality it can open a gap. As a result, a Charge Density Wave (CDW) phase arises in Weyl semi-metal. In this paper was considered a system with two Weyl nodes at $\boldsymbol{K_0}$ (labeled R) and $-\boldsymbol{K_0}$ (labeled L). The conduction (valence) band at the R node has its spin parallel (antiparallel) to $\boldsymbol{k} \mp \boldsymbol{K_0}$ and it's the opposite at the L node. The corresponding Hamiltonian

$$\widehat{H}_{0\pm} = \pm\hbar v_F \sum_{\boldsymbol{k}} \psi^{\dagger}_{\boldsymbol{k}\alpha} \boldsymbol{\sigma}_{\alpha\beta}(\boldsymbol{k} \mp \boldsymbol{K_0})\psi_{\boldsymbol{k}\beta}, \tag{0.10}$$

where v_F is the Fermi velocity and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a vector of Pauli matrices.

The general particle-particle interaction, in momentum space, takes the form

$$\widehat{V} = \sum_{\sigma,\sigma'} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} V(\boldsymbol{q}) \psi^{\dagger}_{\boldsymbol{k}'+\boldsymbol{q},\sigma'} \psi_{\boldsymbol{k}'\sigma'} \psi^{\dagger}_{\boldsymbol{k}-\boldsymbol{q},\sigma} \psi_{\boldsymbol{k}\sigma}. \tag{0.11}$$

Thus, it was found that the energy spectrum of Weyl semi-metals in the bulk is

$$E_{\pm}(p) = \pm\sqrt{(\hbar v_F(p - p_F))^2 + \Delta^2}, \tag{0.12}$$

where Δ is the magnitude of the order parameter of the CDW phase.

It is worth noting the recent experimental paper [6], in which evidence for the existence of CDW phase in $(TaSe_4)_2I$ was reported. Also, the authors found that owing to its quasi-1D structure (see Fig. 2), in this material arises CDW just below room temperature and it couples the bulk Weyl points and opens a band gap.

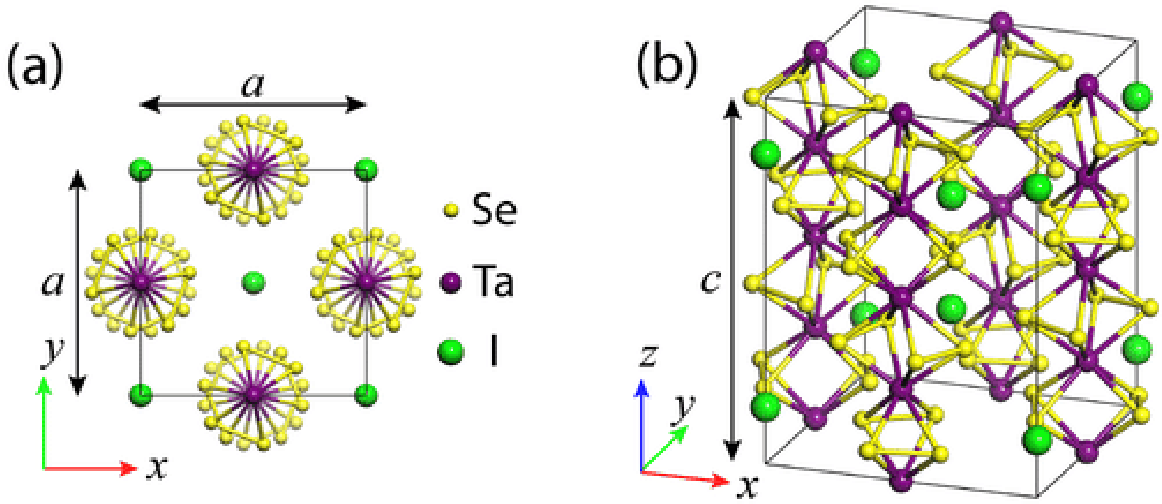

Figure 2: Adopted from work [6]. The crystal structure of the $[(TaSe_4)_2I]$.

Before proceeding to the formulation of the problem, it is worth considering the situation when a Weyl semimetal in the metallic phase in the presence of a magnetic field. It was shown [7] that in the presence of the magnetic field perpendicular to the surface of the crystal, closed magnetic orbits arise (see Fig. 3). Magnetic field leads to Lorentz force acting on the surface electrons that make them slide along the top-surface Fermi arc. In the presence of the magnetic field, the spectrum of bulk states displays Landau levels for $n \neq 0$ and the linear dispersion law for $n = 0$ (chiral Landau level of a Weyl node). At low energies, only the chiral Landau level is available. Therefore, we expect the electron at the tip of the arc will tunnel into the bulk and this chiral Landau level will propagate the electron towards the bottom surface. After that, the electron proceeds to move along the opposite Fermi arc and returns to the top surface along with the oppositely propagating Landau level of the other Weyl node (see Fig. 3). In the article [8] evidence for the existence of discussed above cyclotron trajectories was reported.

However, in the CDW phase of the Weyl semimetal, the situation is different. On the one hand, the surface conducting layer is preserved. As before, in the presence of an external mag-

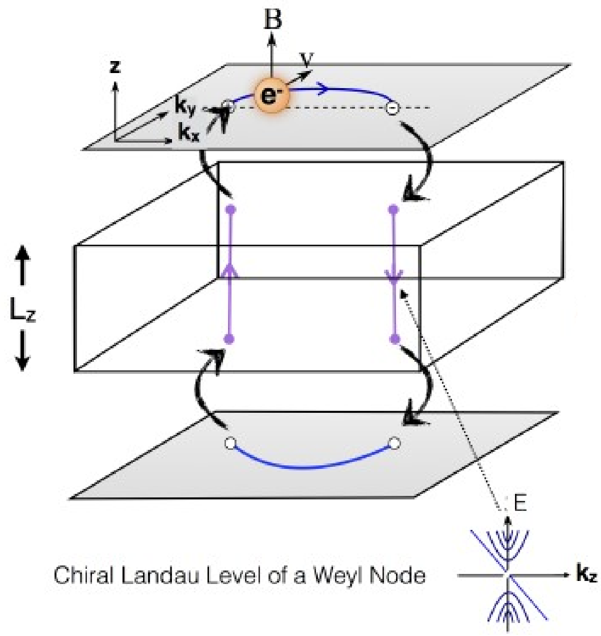

Figure 3: Adopted from work [9]. Closed magnetic orbits in the Weyl semimetal in the presence of the magnetic field along z axis. The picture below shows the Landau levels.

netic field along z axis, the Lorentz force acts on the surface electrons making them move along the Fermi arc. On the other hand, the formation of CDW turns the Weyl semimetal into a bulk insulator, such cyclotron motion is blocked by the bulk gap. Instead, the electron is scattered off the CDW and as a result, it turns in the valley of the Weyl node with opposite chirality. After that, the particle moves along the opposite Fermi arc and repeats the process with another pair of Weyl nodes. Therefore, peculiar cyclotron motion arises. In this project, we study the consequences of the transition to the CDW insulating phase on galvanomagnetic phenomena. The main problem of this project is the computing of the period of cyclotron motion of the Weyl electron described above. We find that the surface cyclotron motion inherits some of the features of the peculiar bulk cyclotron trajectories that exist in the conducting phase. In particular, the period of the surface cyclotron trajectories diverges near the transition between the CDW and metallic phases.

The structure of the presentation is the following. In section 1 we obtain the equation for the surface state electron dispersion. Then we simplify it in the limit of a small value of gap and obtain a very compact formula for the arc state electron dispersion in the leading order in value of the CDW gap. Section 2 contains the calculation of the period of cyclotron motion in the CDW phase of Weyl semi-metal.

1 Spectrum of surface states

We start from the studying of the surface state electron dispersion. This problem may be described by the following Hamiltonian

$$\widehat{H} = (\boldsymbol{\sigma p} - 1)\widehat{\tau}_3 + \Delta\widehat{\tau}_1, \tag{1.1}$$

where $\widehat{\tau}_1, \widehat{\tau}_3$ are Pauli matrices in the chiral representation, Δ is the dimensionless absolute value of the order parameter, $\boldsymbol{\sigma}$ is a vector of Pauli matrices in standard representation, $\boldsymbol{p}$ is a dimensionless momentum of the particle. In this Hamiltonian we choose the energy difference between two nodes is equal to one and all terms are measured in the units of this value.

Wave function which is the general decision for the corresponding Schrodinger equation has the following form

$$\boldsymbol{\Psi} = \begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{w} \end{pmatrix}, \boldsymbol{u} = \begin{pmatrix} u \\ v \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} w \\ h \end{pmatrix}. \tag{1.2}$$

Because of vanishing the normal component of the current the boundary conditions can be presented in the form

$$\begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} e^{i\chi_1} & 0 \\ 0 & e^{i\chi_2} \end{pmatrix} \begin{pmatrix} v \\ h \end{pmatrix}_{z=0}, \tag{1.3}$$

where χ_1, χ_2 are the corresponding phases for each valley.

Using the definition of Pauli matrices and the momentum $\boldsymbol{p} = (q_x, q_y, -i\partial_z)$ we obtain the equation

$$\partial_z \boldsymbol{\Psi} = -\widehat{\kappa}(\varepsilon)\boldsymbol{\Psi}, \quad \widehat{\kappa}(\varepsilon) = \begin{pmatrix} -i\,(\varepsilon+1) & iq^* & i\Delta & 0 \\ -iq & i\,(\varepsilon+1) & 0 & -i\Delta \\ -i\Delta & 0 & i\,(\varepsilon-1) & iq^* \\ 0 & i\Delta & -iq & -i\,(\varepsilon-1) \end{pmatrix}, \tag{1.4}$$

where $q = q_x + iq_y$.

The relevant solution is given by

$$\boldsymbol{\Psi}(z) = e^{-\widehat{\kappa}(\varepsilon)z}\boldsymbol{\Psi}(0), \quad \boldsymbol{\Psi}(0) = \begin{pmatrix} v\exp(i\chi_1) \\ v \\ h\exp(i\chi_2) \\ h \end{pmatrix}. \tag{1.5}$$

Here and afterward, we choose the brunch cut along the negative real axis and use the parametrization of energy in the form $\varepsilon = \Delta\cos\gamma, \gamma \in (0, \pi)$. In this case, we calculate the eigenvalues and eigenvectors of $\widehat{\kappa}(\varepsilon)$. Wave function should decay exponentially at $z \to \infty$ that is why we take eigenvalues with the positive real part and two corresponding eigenvectors. The relevant decision of the equation in the point $z = 0$ can be represented by the linear combination of these two eigenvectors. This yields the equation for the surface state electron dispersion in the form (see Appendix A)

$$\left(\frac{|q|e^{i(\chi_2+\phi)}-\left(1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\right)}{1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}-|q|e^{i(\chi_2+\phi)}}e^{-i\gamma}+e^{i\gamma}\right)e^{i\chi_1}=$$

$$=\frac{|q|e^{i(\chi_2+\phi)}-\left(1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\right)}{1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}-|q|e^{i(\chi_2+\phi)}}\times$$
$$\times\frac{\exp\left(-i\gamma\right)\left(1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}\right)}{|q|e^{i\phi}}+$$
$$+\frac{\exp\left(i\gamma\right)\left(1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\right)}{|q|e^{i\phi}}. \tag{1.6}$$

1.1 CDW bound states

The spectrum of surface states is given by the equation (1.6) for momentum $|q|<1$ and $|q|>1$. We consider the value of the CDW gap small compared to the Fermi energy. In this case, if we consider the momentum region $|q|>1$, we can neglect Δ in the expressions for the eigenvalues (see. (A.2)). Therefore, this situation is similar to the case without CDW. The corresponding solution will be a decaying exponent as you can see from (1.5). Here and afterward, we will refer to these states corresponding to a given momentum region as arc states for the convenience of designations. When moving to the momentum region |q| <1, imaginary parts will appear in the eigenvalues of the matrix $\widehat{\kappa}$ given by (1.4). Thus, an oscillating part appears in the solution. In this case, $1-|q|^2$ corresponds to the oscillating part, and $\frac{\Delta}{\sqrt{1-|q|^2}}$ corresponds to attenuation. The momentum region $1-|q|^2\sim\Delta$ is not important for further calculations of the cyclotron motion period, since this is a very narrow region compared to the one under consideration. In addition, the attenuation in this region will have a greater impact than in the region $1-|q|^2\gg\Delta$. Therefore, we consider the momentum region in which the wave function will penetrate deep into the bulk (the penetration depth is inversely proportional to the Δ in dimensionless quantities) and give the main contribution to the period of oscillations. Here and afterward, we will refer to these states corresponding to a given momentum region as CDW bound states for the convenience of designations.

In the Eq. (1.6) all terms with χ_1,χ_2,ϕ contains the following sums $\chi_1+\phi,\chi_2+\phi$. In this case we can take $\chi_1=-\chi_2$ and measure ϕ from the -$\frac{\chi_1+\chi_2}{2}$. We lead to the common denominator the left and right terms and it yields the equation in the form (see Appendix A)

$$\begin{gathered}-2i\sin\gamma|q|^2e^{i\phi}+4i|q|e^{i\phi}\sin\gamma\cos\chi_1+4\Delta\sin\gamma\cos\gamma|q|e^{i\phi}\sin\chi_1+\\+\left(2i|q|e^{i\phi}\sin\gamma\cos\chi_1-2i\sin\gamma\right)(z_1-z_2)-\\-\left(2i|q|e^{i\phi}\cos\gamma\sin\chi_1+2\Delta\sin^2\gamma\right)(z_1+z_2)=\\=2i\sin\gamma+2i\Delta^2\sin^3\gamma-2i\sin\gamma z_1z_2,\end{gathered} \tag{1.7}$$

where $z_1=\sqrt{1-\Delta^2\sin^2\gamma-|q|^2+2i\Delta\sin\gamma}$, $z_2=\sqrt{1-\Delta^2\sin^2\gamma-|q|^2-2i\Delta\sin\gamma}$.

In the case $1-|q|^2 \gg \Delta$ the Eq. (1.7) can be drastically simplified (the essence of this approximation explained above). In this limit z_1, z_2 are given by

$$z_1 \approx \sqrt{1-|q|^2}\left(1+\frac{i\Delta\sin\gamma}{1-|q|^2}\right),$$
$$z_2 \approx \sqrt{1-|q|^2}\left(1-\frac{i\Delta\sin\gamma}{1-|q|^2}\right). \tag{1.8}$$

To the leading order in the small value of the CDW gap, we obtain the following equation for the surface state electron dispersion

$$\sin\gamma\left(\cos\chi_1-\cos\phi|q|\right)=\cos\gamma\sin\chi_1\sqrt{(1-|q|^2)}. \tag{1.9}$$

This equation describes the surface state electron dispersion in the area where the electron is scattered off the CDW. The constant energy cuts of spectrum given by this equation are shown in Fig. 4.

Besides, it is worth nothing that the obtained equation (1.9) defines an ellipse (see Appendix B). This will be used in the next part of our work to compute the period of cyclotron motion of an electron in the CDW phase of Weyl semimetals.

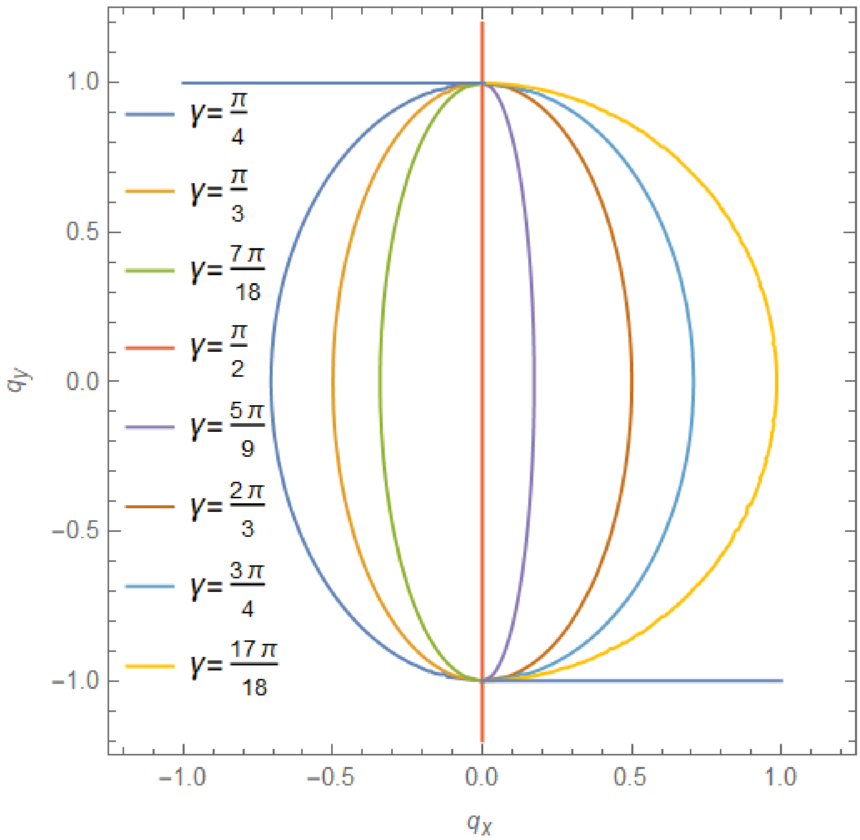

Figure 4: Constant energy cuts of spectrum given by Eq. (1.9) for different values of γ. This plot for the value $\chi_1=\frac{\pi}{2}$. The straight lines in this graph schematically show the transition to Fermi arcs.

2 Period of cyclotron motion in the CDW phase of Weyl semimetals

We want to compute the period of cyclotron motion in the CDW phase of Weyl semimetals. In this case, we use a semiclassical approach (we follow the book [10]). We start with the equation describing the motion of the electron in the presence of the external magnetic field $\boldsymbol{B}$

$$\frac{d\boldsymbol{k}}{dt} = -\frac{e}{c}\left[\dot{\boldsymbol{r}} \times \boldsymbol{B}\right]. \tag{2.1}$$

We assume that the magnetic field is applied along the z direction. Thus, we get for the z - component of momentum $k_z = const$. Also, for the energy we have $\varepsilon = const$. It means that there is trajectory in momentum space as result of intersection of surface $\varepsilon(\boldsymbol{p}) = const$ and plane $p_z = const$. For the x, y - components the equations are given by

$$\frac{dp_x}{dt} = -\frac{e}{c}v_y B, \frac{dp_y}{dt} = \frac{e}{c}v_x B. \tag{2.2}$$

After some simplifications, it yields the formula for the period in the form

$$T = \frac{c}{eB}\oint \frac{dl}{v_\perp}, \tag{2.3}$$

where $dl = \sqrt{dp_x^2 + dp_y^2}$ is the element of length in momentum space and $v_\perp = \sqrt{v_x^2 + v_y^2}$.

On the other hand, we can write for the area closed by the trajectory in the momentum space the following expression

$$S = \int d\varepsilon \oint \frac{dl}{v_\perp}. \tag{2.4}$$

As a result, using (2.3) and (2.4) we get the formula for the period of cyclotron motion in the form

$$T = \frac{c}{eB}\frac{\partial S}{\partial \varepsilon}. \tag{2.5}$$

As discussed above, we have parametrization of energy in the form $\varepsilon = \Delta \cos\gamma$. This yields the formula for the period in the form

$$t = \frac{c}{eB}\left|\frac{\partial S}{\partial \gamma}\frac{\partial \gamma}{\partial \varepsilon}\right| = \frac{c}{eB}\frac{1}{\Delta \sin\gamma}\frac{\mu^2}{v_F^2}\left|\frac{\partial S}{\partial \gamma}\right|, \tag{2.6}$$

where S is the dimensionless value of the area closed by the trajectory in the momentum space, μ is the energy difference between two nodes.

The whole period contains the time of motion along with the areas where the electron is scattered off the CDW and the areas where the electron moves from one Weyl node to another along the surface. In this case, the complete period of cyclotron motion is given by

$$t_{tot} = 2t_{AS} + t_{BS_1} + t_{BS_2}, \tag{2.7}$$

where t_{AS} is respectively the time of the electron motion from one Weyl node to another along the surface or the region where the corresponding states are arc states (as discussed in 1), t_{BS_1} and t_{BS_2} correspond to two regions where the electron is scattered off the CDW or the regions where the corresponding states are CDW bound states in previously accepted designations. In each region, the electron in the valley of one Weyl node interacts with the hole in the valley of the other Weyl node due to the CDW.

Time t_{AS} we can estimate as in the case without a gap in the spectrum. We assume that two Weyl nodes with the opposite chirality are located in the momentum space at $\boldsymbol{k_0}$ and $-\boldsymbol{k_0}$ respectively. Using (2.2) we get the expression for the time in the form

$$t_{AS} \sim \frac{2k_0}{\frac{eB}{c} v_F}, \tag{2.8}$$

where k_0 is a modulus of $\boldsymbol{k_0}$ and v_F is a Fermi velocity. We note that this time doesn't depend on energy and gives a constant contribution to the total period.

To compute the t_{BS_1}, t_{BS_2} we need to find the area swept by the trajectory of the electron in each region. The equation (1.9) defines an ellipse (see Appendix B). Coordinates of the center of this ellipse are given by

$$\left(\frac{\cos\chi_1 \sin^2\gamma}{1 - \cos^2\chi_1 \cos^2\gamma}, 0 \right). \tag{2.9}$$

Focuses of ellipse are given by

$$c = \pm \frac{\sin\chi_1 \sin\gamma}{(1 - \cos^2\chi_1 \cos^2\gamma)}. \tag{2.10}$$

The major axis and minor axis are equal respectively

$$b = \frac{\sin\chi_1}{\sqrt{1 - \cos^2\chi_1 \cos^2\gamma}}, a = \frac{\sin^2\chi_1 |\cos\gamma|}{1 - \cos^2\chi_1 \cos^2\gamma}. \tag{2.11}$$

This ellipse lies inside the unit circle. They intersect in points $(\cos\chi_1, \pm\sin\chi_1)$. If $\gamma = \frac{\pi}{2}$ ellipse degenerates into a straight line $x = \cos\chi_1$. On the interval $\gamma \in (0, \frac{\pi}{2})$ is the relevant solution for this case is the left brunch of ellipse and the right on the interval $\gamma \in (\frac{\pi}{2}, \pi)$. We need to choose respectively in the first case the area between left brunch of ellipse and line $x = \cos\chi_1$ and the area between right brunch of ellipse and corresponding line on the other interval (see Fig. 5).

To compute areas on the different intervals of energy we need to rewrite the ellipse equation in the canonical form (see Appendix B)

$$\frac{(1 - \cos^2\chi_1 \cos^2\gamma)}{\sin^2\chi_1} q_y^2 + \frac{(1 - \cos^2\chi_1 \cos^2\gamma)^2}{\sin^4\chi_1 \cos^2\gamma} \left(q_x - \frac{\cos\chi_1 \sin^2\gamma}{(1 - \cos^2\chi_1 \cos^2\gamma)} \right)^2 = 1. \tag{2.12}$$

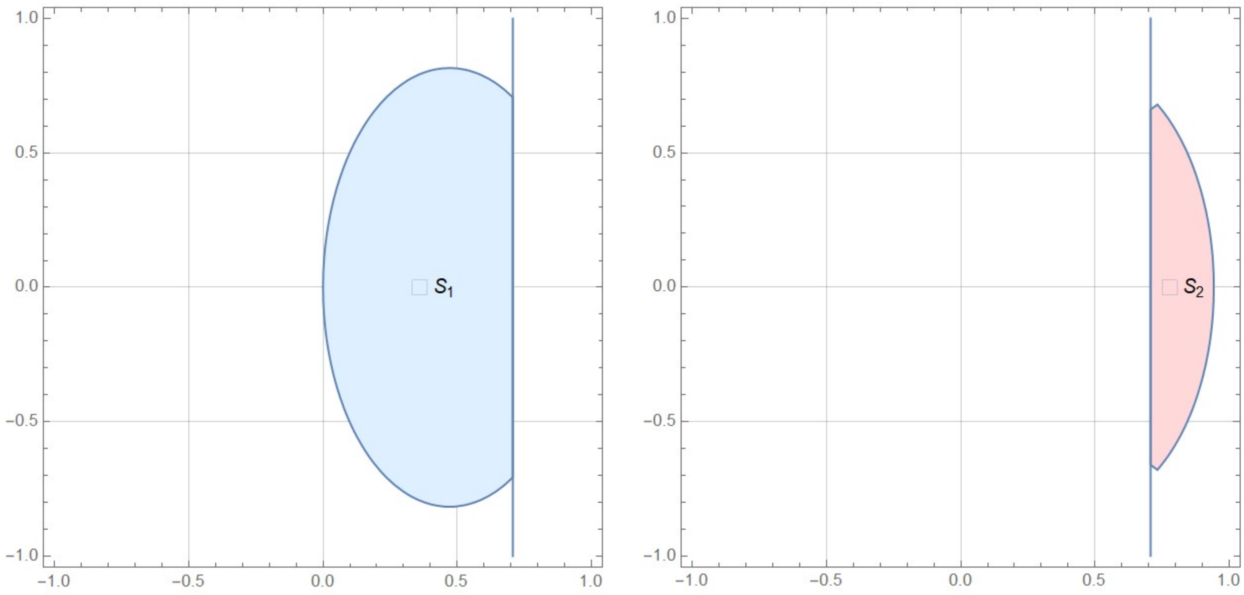

Figure 5: Sketches of areas for the intervals $\gamma \in (0, \frac{\pi}{2})$ and $\gamma \in (\frac{\pi}{2}, \pi)$ respectively. In the first case (left) the parameters are $\chi_1 = \frac{\pi}{4}, \gamma = \frac{\pi}{4}$ and $\chi_1 = \frac{\pi}{4}, \gamma = \frac{3\pi}{4}$ (right).

For the y - component of momentum we get respectively expression in the form

$$q_y^2 = \frac{\sin^2 \chi_1}{(1 - \cos^2 \chi_1 \cos^2 \gamma)} \left(1 - \frac{(1 - \cos^2 \chi_1 \cos^2 \gamma)^2}{\sin^4 \chi_1 \cos^2 \gamma} \left(q_x - \frac{\cos \chi_1 \sin^2 \gamma}{(1 - \cos^2 \chi_1 \cos^2 \gamma)} \right)^2 \right). \tag{2.13}$$

Therefore, using Eq. (2.13), for the interval $\gamma \in (0, \frac{\pi}{2})$ the area is given by the following integral

$$S_1 = 2 \int_{q_1}^{q_2} dq_x \sqrt{\frac{\sin^2 \chi_1}{(1 - \cos^2 \chi_1 \cos^2 \gamma)} \left(1 - \frac{(1 - \cos^2 \chi_1 \cos^2 \gamma)^2}{\sin^4 \chi_1 \cos^2 \gamma} \left(q_x - \frac{\cos \chi_1 \sin^2 \gamma}{(1 - \cos^2 \chi_1 \cos^2 \gamma)} \right)^2 \right)}, \tag{2.14}$$

where limits of integration are given by

$$q_1 = \frac{- \sin^2 \chi_1 \cos \gamma + \cos \chi_1 \sin^2 \gamma}{(1 - \cos^2 \chi_1 \cos^2 \gamma)}, q_2 = \cos \chi_1. \tag{2.15}$$

After calculating of the corresponding integral (see Appendix B) we get

$$S_1 = \frac{\sin^3 \chi_1 \cos \gamma}{(1 - \cos^2 \chi_1 \cos^2 \gamma)^{\frac{3}{2}}} \left(\pi - \arccos(\cos \chi_1 \cos \gamma) + \frac{\sin(2 \arccos(\cos \chi_1 \cos \gamma))}{2} \right). \tag{2.16}$$

We differentiate this expression by the γ

$$\begin{aligned} \frac{\partial S_1}{\partial \gamma} = & - \frac{\sin^3 \chi_1 \sin \gamma (1 + 2 \cos^2 \chi_1 \cos^2 \gamma)}{(1 - \cos^2 \chi_1 \cos^2 \gamma)^{\frac{5}{2}}} \left(\pi - \arccos(\cos \chi_1 \cos \gamma) + \frac{\sin(2 \arccos(\cos \chi_1 \cos \gamma))}{2} \right) + \\ & + \frac{\sin^3 \chi_1 \cos \gamma}{(1 - \cos^2 \chi_1 \cos^2 \gamma)^{\frac{3}{2}}} \frac{\cos \chi_1 \sin \gamma}{\sqrt{1 - \cos^2 \chi_1 \cos^2 \gamma}} \left(\cos(2 \arccos(\cos \chi_1 \cos \gamma)) - 1 \right). \end{aligned} \tag{2.17}$$

As a result, we get the formula for the time on the corresponding interval

$$t_{BS_+} = \frac{c}{eB}\frac{1}{\Delta}\frac{\mu^2}{v_F^2}\times$$
$$\times\left(\frac{\sin^3\chi_1\left(1+2\cos^2\chi_1\cos^2\gamma\right)}{\left(1-\cos^2\chi_1\cos^2\gamma\right)^{\frac{5}{2}}}\left(\pi-\arccos\left(\cos\chi_1\cos\gamma\right)+\frac{\sin\left(2\arccos\left(\cos\chi_1\cos\gamma\right)\right)}{2}\right)-\right.$$
$$\left.-\frac{\sin^3\chi_1\cos\gamma}{\left(1-\cos^2\chi_1\cos^2\gamma\right)^{\frac{3}{2}}}\frac{\cos\chi_1}{\sqrt{1-\cos^2\chi_1\cos^2\gamma}}\left(\cos\left(2\arccos\left(\cos\chi_1\cos\gamma\right)\right)-1\right)\right). \tag{2.18}$$

To compute time on the interval $\gamma\in\left(\frac{\pi}{2},\pi\right)$ we subtract from the whole area of the ellipse S_1. The area of the ellipse is given by

$$S=\pi ab. \tag{2.19}$$

Using (2.11) we get

$$S=\frac{\pi\sin^3\chi_1|\cos\gamma|}{\left(1-\cos^2\chi_1\cos^2\gamma\right)^{\frac{3}{2}}}. \tag{2.20}$$

$$S_2=\frac{\sin^3\chi_1|\cos\gamma|}{\left(1-\cos^2\chi_1\cos^2\gamma\right)^{\frac{3}{2}}}\left(\arccos\left(\cos\chi_1|\cos\gamma|\right)-\frac{\sin\left(2\arccos\left(\cos\chi_1|\cos\gamma|\right)\right)}{2}\right). \tag{2.21}$$

$$t_{BS_-} = \frac{c}{eB}\frac{1}{\Delta}\frac{\mu^2}{v_F^2}\times$$
$$\times\left(\frac{\sin^3\chi_1\left(1+2\cos^2\chi_1\cos^2\gamma\right)}{\left(1-\cos^2\chi_1\cos^2\gamma\right)^{\frac{5}{2}}}\left(\arccos\left(\cos\chi_1|\cos\gamma|\right)-\frac{\sin\left(2\arccos\left(\cos\chi_1|\cos\gamma|\right)\right)}{2}\right)+\right.$$
$$\left.+\frac{\sin^3\chi_1|\cos\gamma|}{\left(1-\cos^2\chi_1\cos^2\gamma\right)^{\frac{3}{2}}}\frac{\cos\chi_1}{\sqrt{1-\cos^2\chi_1\cos^2\gamma}}\left(\cos\left(2\arccos\left(\cos\chi_1|\cos\gamma|\right)\right)-1\right)\right). \tag{2.22}$$

The dependence of t_{BS_+} and t_{BS_-} from the energy for the different values of parameter χ_1 is shown on Fig. 6. For clarity, each graph shows the dependence for the t_{BS_+} and t_{BS_-}. During the transition from one interval to another, graphic for the t_{BS_+} continuously passes into the graphic for the t_{BS_-}. Also, as discussed above, our approximation doesn't work on the edges of these intervals. However, we note that these contributions are small.

Let us calculate t_{BS_+} and t_{BS_-} for the different values of parameter γ. For $\gamma=\frac{\pi}{2}$ the values of t_{BS_+} and t_{BS_-} must coincide. Substituting this value of parameter γ into the formulas (2.18),(2.22) we get

$$t_{BS_+}=\frac{c}{eB}\frac{\mu^2}{v_F^2}\frac{\pi}{2\Delta}\sin^3\chi_1=t_{BS_-}. \tag{2.23}$$

For the $\gamma=0$ the expression for the t_{BS_+} is given by

$$t_{BS_+}=\frac{c}{eB}\frac{\mu^2}{v_F^2}\frac{1}{\Delta}\left(\frac{\left(1+2\cos^2\chi_1\right)}{\sin^2\chi_1}\left(\pi-\chi_1+\frac{\sin 2\chi_1}{2}\right)+\frac{\cos\chi_1}{\sin\chi_1}\left(1-\cos 2\chi_1\right)\right). \tag{2.24}$$

After substituting $\chi_1=\pi$ in this formula we get $t_{BS_+}=0$. Similarly we do for t_{BS_-} and value of $\gamma = \pi$, we obtain the expression in the form

$$t_{BS_-} = \frac{c}{eB}\frac{\mu^2}{v_F^2}\frac{1}{\Delta}\left(\frac{(1+2\cos^2\chi_1)}{\sin^2\chi_1}\left(\chi_1 - \frac{\sin 2\chi_1}{2}\right) - \frac{\cos\chi_1}{\sin\chi_1}(1-\cos 2\chi_1)\right). \tag{2.25}$$

After substituting $\chi_1 = 0$ in this formula we get $t_{BS_-} = 0$. These results for t_{BS_+} and t_{BS_-} are consistent with expectations because for these values of the phase χ_1 area swept by the trajectory of the electron is expressed as zero.

In addition, formula for frequency of cyclotron motion is given by

$$\omega = \frac{2\pi}{t_{tot}} = \frac{2\pi}{2t_{AS} + t_{BS_1} + t_{BS_2}}, \tag{2.26}$$

where t_{AS} is given by (2.8), t_{BS_1} and t_{BS_2} are given by (2.18) or (2.22) for the corresponding energy interval. Therefore, the frequency of cyclotron motion depends on energy and the parameter χ_1. The qualitative behavior of frequency of cyclotron motion as a function of energy becomes clear from the Fig. 6. The dependence of the frequency of cyclotron motion on the CDW gap makes it possible to observe cyclotron resonance.

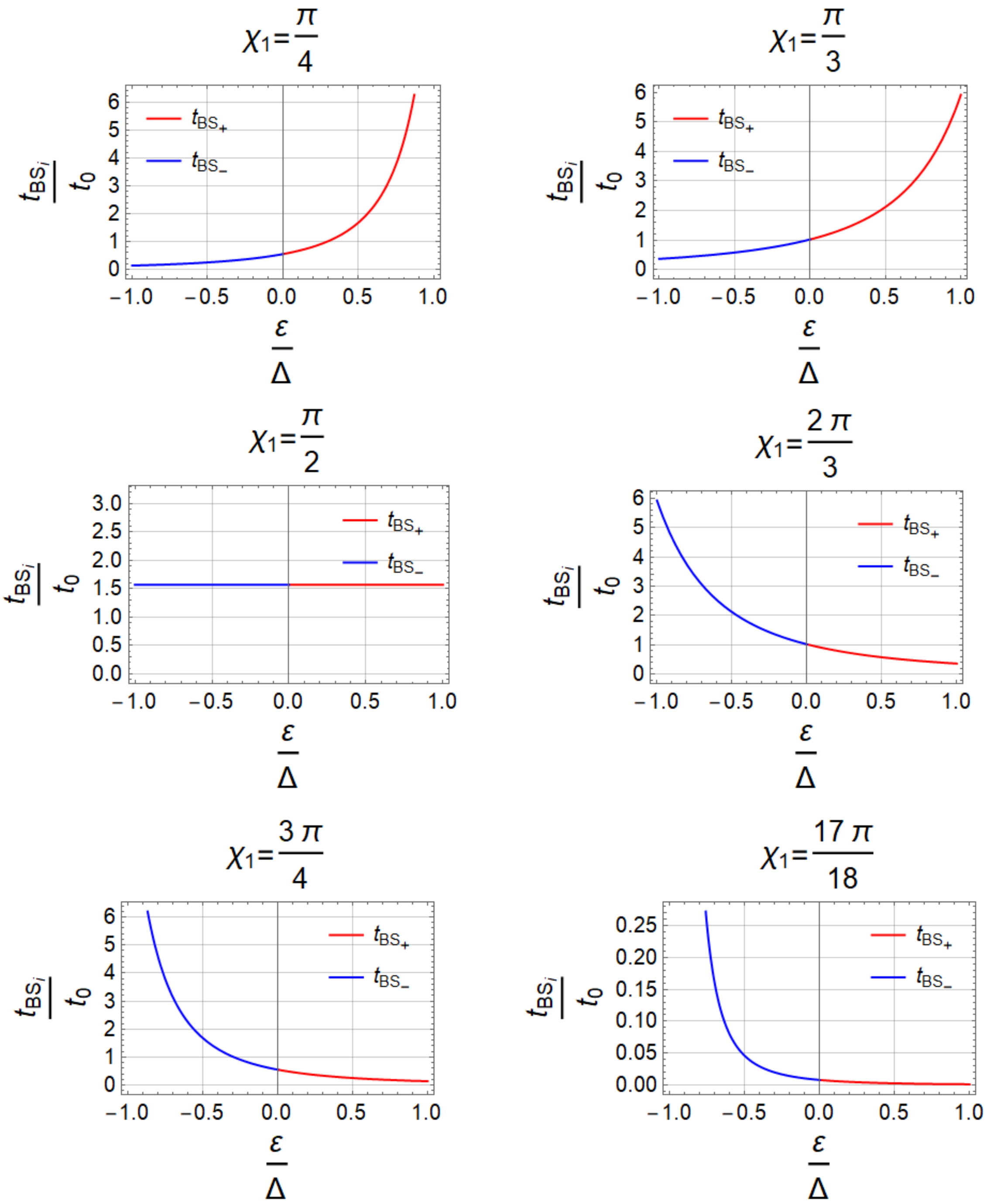

Figure 6: The dependence of t_{BS_+} and t_{BS_-} from the energy for the different values of parameter χ_1. Here we introduced the $t_0 = \frac{l_B^2\mu^2}{\Delta v_F^2}$, where the magnetic length $l_B = \sqrt{\frac{c}{eB}}$ (we consider $\hbar = 1$).

Conclusion

To summarize, we have studied a surface galvanomagnetic phenomena in the CDW phase of Weyl semimetals.

Based on the knowledge that a peculiar periodic motion occurs in the metallic phase of Weyl semimetals in the presence of a magnetic field perpendicular to the crystal surface, we decided to study the consequences of the transition of a Weyl semimetal to the CDW insulating phase. In contrast to the case where the CDW is absent, in our problem the electron does not tunnel into the bulk but is scattered off the CDW. We have obtained a compact formula for the spectrum of the electron surface states in the leading order with respect to the smallness of the gap (see (1.9)). This made it possible to understand that in the situation under consideration, a periodic cyclotron motion of an electron also occurs. In the regions where the momentum is greater than the Fermi momentum, the particle slides along the arc states, and in the regions where the momentum is less than the Fermi momentum, the trajectory of the particle is an ellipse (see Appendix (B.4)). Thus, the full period of the cyclotron motion of the electron is given by the expression (2.7), in which the values of $t_{AS}, t_{BS_1}, t_{BS_2}$ are given by the expressions (2.8), (2.18) or (2.22).

We showed that the value of the period is inversely proportional to the magnitude of order parameter Δ. It means that in these regions where the electron is scattering off the CDW the period of motion slows down. The frequency of cyclotron motion is given by formula (2.26). This frequency is sensitive to changes in the order parameter. This suggests that knowing the temperature dependence of the CDW gap, it is probably possible cyclotron resonance can be measured.

It is worth noting that the obtained expression for the area swept by the electron trajectory makes it possible to find corrections to various quantities arising from Shubnikov–de Haas oscillations.

Appendices

A Derivation of the spectrum of surface states

In this Appendix, we present some details of the derivation of the spectrum of surface states. We find the eigenvalues of $\widehat{\kappa}(\varepsilon)$ in the form

$$\lambda_{1\pm} = \pm\sqrt{-1-\varepsilon^2+\Delta^2+q^2+2\sqrt{\varepsilon^2-\Delta^2}}, \lambda_{2\pm} = \pm\sqrt{-1-\varepsilon^2+\Delta^2+q^2-2\sqrt{\varepsilon^2-\Delta^2}}. \tag{A.1}$$

We choose the brunch cut along the negative real axis. We consider the case $|\varepsilon| < \Delta$ and choose the parametrization of energy in the form $\varepsilon = \Delta\cos\gamma, \gamma \in (0,\pi)$. Then, the corresponding eigenvalues are given by

$$\lambda_{1\pm} = \pm\sqrt{-1+\Delta^2\sin^2\gamma+q^2+2i\Delta\sin\gamma}, \lambda_{2\pm} = \pm\sqrt{-1+\Delta^2\sin^2\gamma+q^2-2i\Delta\sin\gamma}. \tag{A.2}$$

The eigenvalues $\lambda_{1+}, \lambda_{2+}$ have a positive real parts for both cases such as $|q| < 1, |q| > 1$. It means that they obey the condition that the corresponding wave function should decay exponentially at $z \to \infty$. The corresponding eigenvectors are expressed as

$$\xi_1 = \begin{pmatrix} \frac{q^2+\left(-1+i\Delta\sin\gamma-i\sqrt{-1+\Delta^2\sin^2\gamma+q^2+2i\Delta\sin\gamma}\right)\left(1-\Delta\cos\gamma-i\sqrt{-1+\Delta^2\sin^2\gamma+q^2+2i\Delta\sin\gamma}\right)}{q\Delta} \\ e^{-i\gamma} \\ \frac{1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2+2i\Delta\sin\gamma}}{q} \\ 1 \end{pmatrix},$$

$$\xi_3 = \begin{pmatrix} \frac{q^2+\left(1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2-2i\Delta\sin\gamma}\right)\left(-1-\Delta\cos\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2-2i\Delta\sin\gamma}\right)}{q\Delta} \\ e^{i\gamma} \\ \frac{1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2-2i\Delta\sin\gamma}}{q} \\ 1 \end{pmatrix}. \tag{A.3}$$

The relevant decision of the equation in the point $z = 0$ can be represented by the linear combination of these two eigenvectors. After some simplifications, this yields the system of equations

$$\begin{cases} ve^{i\chi_1} = \alpha\frac{e^{-i\gamma}\left(1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2+2i\Delta\sin\gamma}\right)}{q} + \beta\frac{e^{i\gamma}\left(1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2-2i\Delta\sin\gamma}\right)}{q}, \\ v = \alpha e^{-i\gamma} + \beta e^{i\gamma}, \\ he^{i\chi_2} = \alpha\frac{1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2+2i\Delta\sin\gamma}}{q} + \beta\frac{1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+q^2-2i\Delta\sin\gamma}}{q}, \\ h = \alpha+\beta. \end{cases} \tag{A.4}$$

Then, we obtain from this system the equation (1.6).

A.1 CDW bound states

Let us consider the equation (1.6). We can lead to a common denominator left-hand and right-hand sides of the equation and simplify it. The numerator of the left-hand side of the equation is given by

$$\begin{gathered}\left(|q|e^{i(\phi+\chi_2)}-1-i\Delta\sin\gamma-i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\right)e^{-i\gamma}+\\+\left(-|q|e^{i(\phi+\chi_2)}+1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}\right)e^{i\gamma}=\\=-2i\sin\gamma|q|e^{i(\phi+\chi_2)}+2i\sin\gamma-2i\Delta\sin\gamma\cos\gamma-\\-ie^{-i\gamma}\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}+ie^{i\gamma}\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}.\end{gathered}\tag{A.5}$$

The numerator of the right-hand side of equation is given by

$$\begin{gathered}\left(|q|e^{i(\phi+\chi_2)}-1-i\Delta\sin\gamma-i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\right)\times\\\times e^{-i\gamma}\left(1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}\right)+\\+\left(-|q|e^{i(\phi+\chi_2)}+1-i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}\right)\times\\\times e^{i\gamma}\left(1+i\Delta\sin\gamma+i\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\right)=\\=-2i\sin\gamma|q|e^{i(\phi+\chi_2)}-2i\Delta\sin\gamma\cos\gamma|q|e^{i(\phi+\chi_2)}+\\+\left(ie^{-i\gamma}|q|e^{i(\phi+\chi_2)}-ie^{-i\gamma}+\Delta\sin\gamma e^{-i\gamma}+ie^{i\gamma}-\Delta\sin\gamma e^{i\gamma}\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}\right)+\\+\left(-ie^{i\gamma}|q|e^{i(\phi+\chi_2)}+ie^{i\gamma}+\Delta\sin\gamma e^{i\gamma}-ie^{-i\gamma}-\Delta\sin\gamma e^{-i\gamma}\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\right)+\\+2i\sin\gamma+2i\Delta\sin\gamma\cos\gamma-2i\Delta\sin\gamma\cos\gamma+2i\Delta^2\sin^3\gamma-\\-2i\sin\gamma\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}.\end{gathered}\tag{A.6}$$

After we take $\chi_1=-\chi_2$ and equate both parts of the equation, we get the equation in the form

$$\begin{gathered}-2i\sin\gamma|q|^2e^{2i\phi}+2i|q|e^{i(\chi_1+\phi)}\sin\gamma(1-\Delta\cos\gamma)+\\+\left(-2|q|e^{i\phi}\sin\gamma-\chi_1+2\sin\gamma-2i\Delta\sin^2\gamma\right)\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}+\\+\left(-2|q|e^{i\phi}\sin\gamma+\chi_1+2\sin\gamma+2i\Delta\sin^2\gamma\right)\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}=\\=-2i\sin\gamma|q|e^{i(\phi-\chi_1)}-2i\Delta\sin\gamma\cos\gamma|q|e^{i(\phi-\chi_1)}+2i\sin\gamma+2i\Delta^2\sin^3\gamma-\\-2i\sin\gamma\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2-2i\Delta\sin\gamma}\sqrt{-1+\Delta^2\sin^2\gamma+|q|^2+2i\Delta\sin\gamma}.\end{gathered}\tag{A.7}$$

This equation can be easily simplified to the form of Eq. (1.7).

B Derivation of the period of cyclotron motion

In this Appendix, we present some details of the derivation of the period of cyclotron motion.

B.1 Analysis of the ellipse equation

Let us consider the Eq. (1.9) in detail. We can rewrite this equation in the form

$$\sin\gamma\left(\cos\chi_1 - q_x\right) = \cos\gamma\sin\chi_1\sqrt{1-q_x^2-q_y^2}. \tag{B.1}$$

After squaring this expression and some simplifications we get the equation

$$-\left(\cos^2\chi_1 - 2\cos\chi_1 q_x + q_x^2\right) + \left(1 + q_x^2 - 2\cos\chi_1 q_x\right)\cos^2\gamma = \cos^2\gamma\sin^2\chi_u\left(q_x^2+q_y^2\right) \tag{B.2}$$

We can express q_y from this equation. It yields the formula in the form

$$\begin{aligned} q_y^2 + \frac{1}{\sin^2\chi_1\cos^2\gamma}\left(q_x\sqrt{1-\cos^2\chi_1\cos^2\gamma} - \frac{\cos\chi_1\sin^2\gamma}{\sqrt{1-\cos^2\chi_1\cos^2\gamma}}\right)^2 = \\ = \frac{1}{\sin^2\chi_1\cos^2\gamma}\left(\frac{\cos^2\chi_1\sin^4\gamma}{1-\cos^2\chi_1\cos^2\gamma} + \cos^2\gamma - \cos^2\chi_1\right). \end{aligned} \tag{B.3}$$

After simplification of the expression on the right-hand side of the equation, the ellipse equation in the canonical form is given by

$$\frac{\left(1-\cos^2\chi_1\cos^2\gamma\right)}{\sin^2\chi_1}q_y^2 + \frac{\left(1-\cos^2\chi_1\cos^2\gamma\right)^2}{\sin^4\chi_1\cos^2\gamma}\left(q_x - \frac{\cos\chi_1\sin^2\gamma}{\left(1-\cos^2\chi_1\cos^2\gamma\right)}\right)^2 = 1. \tag{B.4}$$

To calculate the area closed by the trajectory, we need to find the coordinates of points where the ellipse intersects with the unit circle. We substitute $|q| = 1$ in the Eq. (1.9) and we get

$$q_x = \cos\chi_1, q_y = \pm\sin\chi_1. \tag{B.5}$$

Also, for further integration we need to compute coordinates of points where the ellipse intersects the x - axis. Substituting in the Eq. (B.4) the value $q_y = 0$ we find corresponding coordinates

$$q_x = \frac{\pm\sin^2\chi_1|\cos\gamma| + \cos\chi_1\sin^2\gamma}{\left(1-\cos^2\chi_1\cos^2\gamma\right)}. \tag{B.6}$$

B.2 Computing of the area

For the area swept by the trajectory of the electron for the energies in the interval $(0, \frac{\pi}{2})$ we have the formula (2.14). In this integral, we replace the integration variable as follows

$$q_x \to x + \frac{\cos\chi_1 \sin^2\gamma}{(1-\cos^2\chi_1\cos^2\gamma)}. \tag{B.7}$$

In this case limits of integration become

$$x_1 = -\frac{\sin^2\chi_1\cos\gamma}{1-\cos^2\chi_1\cos^2\gamma}, x_2 = \frac{\cos\chi_1\sin^2\chi_1\cos^2\gamma}{1-\cos^2\chi_1\cos^2\gamma}. \tag{B.8}$$

The expression for the S_1 is given by

$$S_1 = \frac{2\sin\chi_1}{\sqrt{1-\cos^2\chi_1\cos^2\gamma}}\int\limits_{x_1}^{x_2} dx\sqrt{1-\frac{(1-\cos^2\chi_1\cos^2\gamma)^2}{\sin^4\chi_1\cos^2\gamma}x^2}. \tag{B.9}$$

After that, we make two consecutive substitutions in the form

$$x \to \frac{\sin^2\chi_1\cos\gamma}{(1-\cos^2\chi_1\cos^2\gamma)}y, y \to \cos z. \tag{B.10}$$

It corresponds to the changing of limits of integration as follows

$$\begin{gathered} x_1 \to y_1 = -1, x_2 \to y_2 = \cos\chi_1\cos\gamma, \\ y_1 \to z_1 = \pi, y_2 \to z_2 = \arccos(\cos\chi_1\cos\gamma). \end{gathered} \tag{B.11}$$

Finally, we get the following expression for the S_1

$$S_1 = -\frac{\sin^3\chi_1\cos\gamma}{(1-\cos^2\chi_1\cos^2\gamma)^{\frac{3}{2}}}\int\limits_{\pi}^{\arccos(\cos\chi_1\cos\gamma)} dz\,(1-\cos 2z)\,. \tag{B.12}$$

After computing this integral we obtain (2.16). Calculations for the S_2 are similar.

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