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МЕЗОСКОПИЧЕСКИЕ ФЛУКТУАЦИИ ЛОКАЛЬНОЙ ПЛОТНОСТИ СОСТОЯНИЙ В ГРЯЗНОМ СВЕРХПРОВОДНИКЕ С РЕДКИМИ МАГНИТНЫМИ ПРИМЕСЯМИ

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MASTER'S THESIS

Fluctuations of the local density of states below superconducting transition in the presence of magnetic impurities

Master's Educational Program: Mathematical and Theoretical Physics

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Mathematical and Theoretical Physics
June 18, 2021

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Moscow 2021

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МАГИСТЕРСКАЯ ДИССЕРТАЦИЯ

Флуктуации локальной плотности состояний ниже сверхпроводящего перехода в присутствии магнитных примесей

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Fluctuations of the local density of states below superconducting transition in the presence of magnetic impurities

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Abstract

In superconductors, presence of magnetic impurities creates a pair-breaking potential for Cooper pairs, which affects the density of states (DOS).The mean-field theory predicts a nontrivial structure of the DOS with a continuum of quasiparticle states and a possible impurity band. We investigate a superconductive system with magnetic impurities within the replica sigma-model formalism, we calculate fluctuations of local density of the states (LDOS) to understand if these fluctuations can destroy self-averaging of DOS.

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Флуктуации локальной плотности состояний ниже сверхпроводящего перехода в присутствии магнитных примесей

Пашинский Борис Витальевич

Представлено в Сколковский институт науки и технологий
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Реферат

В сверхпроводниках наличие магнитных примесей частично разрушает куперовские пары, тем самым влияет на плотность состояний. Теория среднего поля предсказывает нетривиальную структуру плотности состояний с континуумом состояний квазичастиц и возможной примесной зоной. Рассматривая сверхпроводимость с магнитными примесями в репличной технике сигма-модели, мы рассчитали флуктуации локальной плотности состояний, чтобы понять, могут ли эти флуктуации разрушить самоусреднение плотности состояний.

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Contents

Chapter 1

INTRODUCTION

In s-wave superconductors, potential impurities do not change the density of the states (DOS) [1]. But magnetic impurities that violate the time-reversal symmetry influence superconductivity much more strongly [2, 3]. Partial destruction of superconductivity by magnetic impurities leads to a change in the DOS which, using the mean field method, was studied by Abrikosov and Gor'kov [4]. In the works [5, 6], the fluctuation smearing of the gap edges was shown, which proves the importance of fluctuations for DOS. Therefore, the study of fluctuations in the local densities of the states (LDOS) is chosen as the research subject of this thesis.

It is also worth mentioning that low-dimensional superconductors with magnetic impurities can have zero Majorana modes. These quasiparticles can be used to build topologically protected qubits, which can make quantum computing more robust to computational errors [7, 8, 9].

In the first chapter, we give the motivation for this work. We discuss interaction between the superconductors and the magnetic impurities as well as formulation of the problem. The chapter two contains calculations made by replica sigma-model method that lead us to the propagator of densities of the states. The third chapter has derivation of DOS correction. The fourth chapter tells about finding fluctuations of LDOS. In the fifth chapter will be discussion of received results.

1.1 Motivation

As stated in the 1, DOS is modified in the presence of magnetic impurities. One of the typical forms of DOS dependence is shown in the Fig. 1.1, as you can see, in contrast to BCS, the DOS

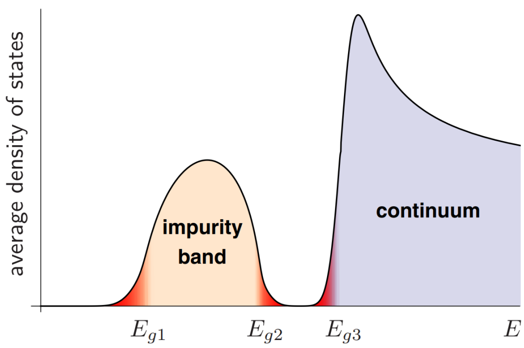

Figure 1.1: One of schematic forms of the average DOS (adopted from work [10])

in the presence of magnetic impurities has a so-called impurity band and there is no hard edge of spectrum. Fig. 1.2 shows all possible DOS dependencies depending on parameters μ and η that are defined as:

$$\mu = \frac{2\alpha}{1+\alpha^2}, \alpha = (\pi\nu JS)^2 \tag{1.1}$$

$$\eta = \frac{n_s \mu}{\pi\nu\Delta} \tag{1.2}$$

where J is energy parameter characterizing of magnetic (spin-flip) scattering, ν is the DOS at the Fermi energy per one spin projection, S is the spin and $\bar{n_s}$ is the average concentration of magnetic impurities, which are distributed in accordance with Poisson distribution.

It was shown in the work [11] that there is a nonzero DOS for energies $E < \Delta$ where Δ denotes superconductor energy gap, for a superconductor with magnetic impurities, in contrast to a superconductor without impurities. At the Fermi level, $E = 0$, the DOS is exponentially small. In this context, it becomes interesting is there energy when fluctuations of the density of states become larger than the average value and self-averaging of DOS breaks down? For the case without magnetic impurities, the answer is known from [12]: $\frac{\langle [\rho(E,r) - \langle \rho(E,r)\rangle]^2 \rangle}{\langle \rho(E,r)\rangle^2} = \frac{4}{\pi g} \ln L/l$, where $\rho(E,r)$ is LDOS, g and L are the bare dimensionless conductance and length of the film, l is the mean-level spacing. Thus, the main question that this work answers is how this expression will look if we consider a superconductor with magnetic impurities and whether the fluctuations of LDOS will be greater than the magnitude of DOS.

1.2 Problem statement

To find fluctuations we need to formulate action in terms of replica nonlinear sigma-model(NLSM). In our case it will be look like[13]:

$$S = S_\sigma + S_{int}^{(c)} + S_{mag} \tag{1.3}$$

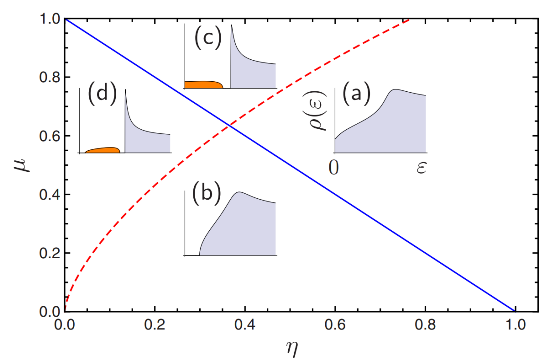

Figure 1.2: Classification of various forms of the average DOS (adopted from work [10])

$$S_\sigma = -\frac{g}{32}\int d\boldsymbol{r}\,\mathrm{Tr}\,(\nabla Q)^2 + 2Z_\omega \int d\boldsymbol{r}\,\mathrm{Tr}\,\hat{\varepsilon} Q \tag{1.4}$$

$$S_{int}^{(c)} = \frac{4Z_\omega}{\pi T \gamma_c}\int d\boldsymbol{r} \sum_\alpha \sum_{r=1,2} \left[\Delta_r^\alpha(\boldsymbol{r})\right]^2 + 2Z_\omega \int d\boldsymbol{r} \sum_\alpha \sum_{r=1,2} \Delta_r^\alpha(\boldsymbol{r})\,\mathrm{Tr}\, t_{r0} L_0^\alpha Q \tag{1.5}$$

$$S_{mag} = n_s \int d\boldsymbol{r} \left\langle e^{\frac{1}{2}\mathrm{Tr}\ln\left(1+\sqrt{\alpha} Q t_{3j} n_j\right)} - 1 \right\rangle_{\vec{n}} \tag{1.6}$$

To get the magnetic part of the action, we need to average over a unit vector $\vec{n}$. This action contain the following matrices and energies:

$$\Lambda_{nm}^{\alpha\beta} = \mathrm{sgn}\varepsilon_n \delta_{nm}\delta^{\alpha\beta} t_{00},\ (L_k^\gamma)_{nm}^{\alpha\beta} = \delta_{\varepsilon_n+\varepsilon_m,\omega_k}\delta^{\alpha\beta}\delta^{\alpha\gamma} t_{00},\ \omega_k = 2\pi T k, \varepsilon_k = (2k+1)\pi T \tag{1.7}$$

where $\alpha, \beta = 1, ..., N_r$ are replica indices and t_{rj} matrix describe S spin (subscript j) and particle-hole (subscript r) spaces:

$$t_{rj} = \tau_r \otimes s_j,\ r, j = 0, 1, 2, 3 \tag{1.8}$$

where τ_r and s_j are standard Pauli matrices. Q is NLSM matrix field that can be parameterized by matrix field W:

$$Q = R^{-1}(W + \Lambda\sqrt{1 - W^2})R,\ W = \begin{pmatrix} 0 & w \\ \bar{w} & 0 \end{pmatrix} \tag{1.9}$$

Here and afterwards we use the following convention: $W_{n_1 n_2} = w_{n_1 n_2}$ and $W_{n_2 n_1} = \bar{w}_{n_2 n_1}$ with $n_1 > 0, n_2 < 0$, w and $\bar{w}$ have the following (charge-conjugation) symmetry:

$$\bar{w} = t_{12} w^T t_{12},\ w = t_{12} w^* t_{12} \tag{1.10}$$

It is convenient to decompose w as:

$$w_{n_1 n_2}^{\alpha\beta} = \sum_{rj} (w_{n_1 n_2}^{\alpha\beta})_{rj} t_{rj} \tag{1.11}$$

where $(w_{n_1 n_2}^{\alpha\beta})_{rj}$ are completely imaginary or real. The rotation matrices are defined by

$$(R)_{nm}^{\alpha\beta} = \delta^{\alpha\beta}\delta_{nm}\cos(\theta_{\varepsilon_n}/2) + t_{10}\mathrm{sgn}\varepsilon_n \delta^{\alpha\beta}\delta_{\varepsilon_n+\varepsilon_m,0}\sin(\theta_{\varepsilon_n}/2), \tag{1.12}$$

$$\left(R^{-1}\right)_{nm}^{\alpha\beta} = \delta^{\alpha\beta}\delta_{nm}\cos(\theta_{\varepsilon_n}/2) - t_{10}\mathrm{sgn}\varepsilon_n \delta^{\alpha\beta}\delta_{\varepsilon_n+\varepsilon_m,0}\sin(\theta_{\varepsilon_n}/2). \tag{1.13}$$

The parameter Z_ω was introduced by Finkel'stein[14] to describe the renormalization of the frequency term in action (1.3), $\gamma_c = \Gamma_c / Z_\omega$ where Γ_c is singlet Cooper channels interaction parameter. Because we are considering situation close to mean-field saddle point solution we can assume $\Delta_1^\alpha = \Delta$, $\Delta_2^\alpha = 0$. The averaged DOS can be obtained following these expressions [13]:

$$\langle \rho(E,r) \rangle = \mathrm{Re} K_1 \left(i\varepsilon_n \longrightarrow E + i0 \right), \; K_1 \left(i\varepsilon_n \right) = \frac{\rho_0}{4} \mathrm{tr} \left\langle (\Lambda Q)^{\alpha\alpha}_{\varepsilon_n \varepsilon_n} \right\rangle_{S^{(2)}} \tag{1.14}$$

Correlation function of LDOS can be defined[13] using bilenear in Q operator $P_2^{\alpha_1\alpha_2}(i\varepsilon_n, i\varepsilon_m)$:

$$\begin{aligned} K_2(E,r,E',r) = \langle \rho(E',r)\rho(E,r) \rangle - \langle \rho(E',r) \rangle \langle \rho(E,r) \rangle = \\ = \frac{\rho_0^2}{32} \mathrm{Re} \left[P_2^{\alpha_1\alpha_2} \left(i\varepsilon_n, i\varepsilon_m \right)_{i\varepsilon_n = E + i0^+, i\varepsilon_m = E' + i0^+} - P_2^{\alpha_1\alpha_2} \left(i\varepsilon_n, i\varepsilon_m \right)_{i\varepsilon_k = E + i0^+, i\varepsilon_p = E' - i0^+} \right] \end{aligned} \tag{1.15}$$

$$P_2^{\alpha_1\alpha_2}(i\varepsilon_n, i\varepsilon_m) = \langle \langle \mathrm{tr} Q^{\alpha_1\alpha_1}_{nn} \left(r \right) \mathrm{tr} Q^{\alpha_2\alpha_2}_{mm} \left(r \right) \rangle \rangle - 2 \left\langle \mathrm{tr} Q^{\alpha_1\alpha_2}_{nm} \left(r \right) Q^{\alpha_2\alpha_1}_{mn} \left(r \right) \right\rangle, \; \alpha_1, \alpha_2 \neq 0 \tag{1.16}$$

So the problem is to find correlators $\left\langle \left(w^{\alpha\beta}_{\varepsilon_n, -\varepsilon_m} \left(q \right) \right)_{il} \left(\bar{w}^{\gamma\sigma}_{-\varepsilon_k, \varepsilon_p} \left(-q \right) \right)_{il} \right\rangle$ and using it in equations (1.15) and (1.16) get the fluctuation of LDOS. It is also worth to say that Usadel equation in this case would be [11]:

$$-\left| \varepsilon \right| \sin \left(\theta_\varepsilon \right) + \Delta \cos \left(\theta_\varepsilon \right) - \frac{n_s}{4Z_\omega} \frac{\alpha \sin \left(2\theta_\varepsilon \right)}{1 + \alpha^2 + 2\alpha \cos \left(2\theta_\varepsilon \right)} = 0 \tag{1.17}$$

Chapter 2

CALCULATION OF PROPAGATOR USING REPLICA NLSM METHOD

To find fluctuations, we need the second order expansion in $\left(w^{\gamma\alpha}_{n,m}\right)_{pi}$. The expression without magnetic impurities [10]:

$$S^{(2)}_{nonmag} = -\frac{g}{4}\int \frac{d^d q}{(2\pi)^d} \sum_{mn<0,\alpha,\gamma,p,i} \left[q^2 + \frac{1}{D}\left[|\varepsilon_n| \cos\left(\theta_{\varepsilon_n}\right) + \Delta \sin\left(\theta_{\varepsilon_n}\right) + |\varepsilon_m| \cos\left(\theta_{\varepsilon_m}\right) + \right.\right.$$
$$\left.\left. + \Delta \sin\left(\theta_{\varepsilon_m}\right)\right]\right] \left(\overline{w}^{\alpha\gamma}_{m,n}(q)\right)_{pi} \left(w^{\gamma\alpha}_{n,m}(-q)\right)_{pi} \quad (2.1)$$

So to get second order expansion of action we need to decompose only magnetic part (1.6).

2.1 Second order expansion

To decompose only magnetic part we need to separate constant part from (1.6):

$$\operatorname{Tr}\ln\left(1 + i\sqrt{\alpha}Qt_{3j}n_j\right) = \operatorname{Tr}\ln\left(1 + i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j + i\sqrt{\alpha}\left[Q - R^{-1}\Lambda R\right]t_{3j}n_j\right) =$$
$$= \operatorname{Tr}\ln\left(1 + i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right) + \operatorname{Tr}\ln\left(1 + \left[1 + i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}\left[Q - R^{-1}\Lambda R\right]t_{3j}n_j\right) \quad (2.2)$$

To get action we need expansion of exponent, so we get one constant part, influence of this part can be observed in Usadel equation (1.17), there are also the one linear in W part and three quadratic in W parts:

$$e^{\frac{1}{2}\text{Tr}\ln\left(1+i\sqrt{\alpha}Qt_{3j}n_j\right)} = 1 + \frac{1}{2}\text{Tr}\ln\left(1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right) +$$
$$+ \frac{1}{2}\text{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}WRt_{3j}n_j\right) +$$
$$+ \frac{1}{8}\left[\text{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}WRt_{3j}n_j\right)\right]^2 -$$
$$- \frac{1}{4}\text{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}\Lambda\frac{W^2}{2}Rt_{3j}n_j\right) -$$
$$- \frac{1}{4}\text{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}WRt_{3j}n_j\right)^2 + O\left(W^3\right)$$

2.1.1 First part of action

We can calculate trace:

$$\text{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}WRt_{3j}n_j\right) =$$
$$= -\sum_{\varepsilon_n>0,\gamma} \frac{4\sqrt{\alpha}}{1+2\alpha\cos\left[2\theta_{\varepsilon_n}\right]+\alpha^2}\left[i\sin\left(\theta_{\varepsilon_n}\right)(1-\alpha)\sum_{l=1}^{3}\left(\left(w^{\gamma\gamma}_{\varepsilon_n,-\varepsilon_n}\right)_{2j} - \left(\overline{w}^{\gamma\gamma}_{-\varepsilon_n,\varepsilon_n}\right)_{2j}\right)n_j + \right.$$
$$\left. + \sqrt{\alpha}\sin\left[2\theta_{\varepsilon_n}\right]\left(\left(w^{\gamma\gamma}_{\varepsilon_n,-\varepsilon_n}\right)_{10} + \left(\overline{w}^{\gamma\gamma}_{-\varepsilon_n,\varepsilon_n}\right)_{10}\right)\right].$$

And than we can square this trace and average it over unit vector:

$$\frac{1}{8}\left\langle\left[\text{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}WRt_{3j}n_j\right)\right]^2\right\rangle_{\vec{n}} =$$
$$= \sum_{\varepsilon_m>0,\varepsilon_n>0,\gamma,\beta} \frac{2\alpha}{\left(1+2\alpha\cos\left[2\theta_{\varepsilon_m}\right]+\alpha^2\right)\left(1+2\alpha\cos\left[2\theta_{\varepsilon_n}\right]+\alpha^2\right)}\times$$
$$\times\left[-\frac{4}{3}\sin\left(\theta_{\varepsilon_n}\right)\sin\left(\theta_{\varepsilon_m}\right)(1-\alpha)^2\sum_{l=1}^{3}\left(w^{\gamma\gamma}_{\varepsilon_m,-\varepsilon_m}\right)_{2l}\left(\overline{w}^{\beta\beta}_{-\varepsilon_n,\varepsilon_n}\right)_{2l} + \right. \tag{2.3}$$
$$\left. +4\alpha\sin\left[2\theta_{\varepsilon_n}\right]\sin\left[2\theta_{\varepsilon_m}\right]\left(w^{\gamma\gamma}_{\varepsilon_m,-\varepsilon_m}\right)_{10}\left(\overline{w}^{\beta\beta}_{\varepsilon_n,-\varepsilon_n}\right)_{10}\right]$$

2.1.2 Second part of action

Second part involves all the expansions of W in t_{il} matrices:

$$\left\langle \mathrm{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}\Lambda\frac{W^2}{2}Rt_{3j}n_j\right)\right\rangle_{\vec{n}} =$$

$$= 2\sum_{\varepsilon_n>0,\varepsilon_k>0,\gamma,\beta,0\le i,l\le 3}\left(w^{\gamma\beta}_{\varepsilon_n,-\varepsilon_k}\right)_{il}\left(\overline{w}^{\beta\gamma}_{-\varepsilon_k,\varepsilon_n}\right)_{il}\times$$

$$\times\left[\frac{\alpha\left[\cos\left[2\theta_{\varepsilon_n}\right]+\alpha\right]\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)+\alpha\left[\cos\left[2\theta_{\varepsilon_k}\right]+\alpha\right]\left(1+2\alpha\cos\left[2\theta_{\varepsilon_n}\right]+\alpha^2\right)}{\left(1+2\alpha\cos\left[2\theta_{\varepsilon_n}\right]+\alpha^2\right)\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)}\right] \tag{2.4}$$

2.1.3 Third part of action

The third part is more complicated then the other ones but it can be calculated:

$$\left\langle \mathrm{Tr}\left(\left[1+i\sqrt{\alpha}R^{-1}\Lambda R t_{3j}n_j\right]^{-1} i\sqrt{\alpha}R^{-1}WRt_{3j}n_j\right)^2\right\rangle_{\vec{n}} =$$

$$= -\sum_{\varepsilon_n>0,\varepsilon_m>0,\alpha,\beta,0\le i,l\le 3}\frac{4\alpha}{3\left(1+2\alpha\cos\left[2\theta_{\varepsilon_m}\right]+\alpha^2\right)\left(1+2\alpha\cos\left[2\theta_{\varepsilon_n}\right]+\alpha^2\right)}$$

$$\left\{\left[(1+\alpha)^2\cos\left[\theta_{\varepsilon_m}\right]\cos\left[\theta_{\varepsilon_n}\right]\times\left(1-4\delta_{l0}\right)\left(1-2\delta_{i3}-2\delta_{i0}\right)+3\alpha\left(\alpha+\cos\left[2\theta_{\varepsilon_m}\right]\right)\left(\alpha+\cos\left[2\theta_{\varepsilon_n}\right]\right)\right]\times\right.$$

$$\times\left[\left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}\right)_{il}\left(\bar{w}^{\beta\alpha}_{-\varepsilon_m,\varepsilon_n}\right)_{il}+\left(\bar{w}^{\alpha\beta}_{-\varepsilon_n,\varepsilon_m}\right)_{il}\left(w^{\beta\alpha}_{\varepsilon_m,-\varepsilon_n}\right)_{il}\right]+$$

$$+\left[-3\alpha\sin\left[2\theta_{\varepsilon_m}\right]\sin\left[2\theta_{\varepsilon_n}\right]\left(1-2\delta_{i3}-2\delta_{i2}\right)-\left(1-2\delta_{i0}-2\delta_{i2}\right)\left[1-4\delta_{l0}\right](1-\alpha)^2\sin\left[\theta_{\varepsilon_m}\right]\sin\left[\theta_{\varepsilon_n}\right]\right]\times$$

$$\left.\times\left[\left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}\right)_{il}\left(w^{\beta\alpha}_{\varepsilon_m,-\varepsilon_n}\right)_{il}+\left(\bar{w}^{\alpha\beta}_{-\varepsilon_n,\varepsilon_m}\right)_{il}\left(\bar{w}^{\beta\alpha}_{-\varepsilon_m,\varepsilon_n}\right)_{il}\right]\right\} \tag{2.5}$$

2.2 Findings correlators

So quadratic part of action contains contributions without impurities and with impurities:

$$S^{(2)} = S^{(2)}_{\text{nonmag}} + S^{(2)}_{\text{mag}} \tag{2.6}$$

We have quadratic action with three types of terms and we can group it by determining $A_{\varepsilon_n,\varepsilon_m,i,l}$, $B_{\varepsilon_n,\varepsilon_m,i,l}$ and $C_{\varepsilon_n,\varepsilon_m,i,l}$, all three types can be account in matrix $\Phi_{\{\varepsilon_n,\varepsilon_p\},\{\varepsilon_m,\varepsilon_k\},\{\alpha,\sigma\},\{\beta,\gamma\},i,l}$:

$$S^{(2)} = \int \frac{d^dq}{(2\pi)^d} \sum_{\varepsilon_n>0,\varepsilon_m>0,\alpha,\beta,0\le i,l\le 3} \Bigg[\left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}(-q)\right)_{il} A_{\varepsilon_n,\varepsilon_m,i,l} \left(\bar{w}^{\beta\alpha}_{-\varepsilon_m,\varepsilon_n}(q)\right)_{il} +$$
$$+\left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}(-q)\right)_{il} B_{\varepsilon_n,\varepsilon_m,i,l} \left(\overline{w}^{\alpha\beta}_{-\varepsilon_n,\varepsilon_m}(q)\right)_{il} + \left(w^{\alpha\alpha}_{\varepsilon_m,-\varepsilon_m}(-q)\right)_{il} C_{\varepsilon_n,\varepsilon_m,i,l} \left(\overline{w}^{\beta\beta}_{-\varepsilon_n,\varepsilon_n}(q)\right)_{il} \Bigg] =$$
$$= \int \frac{d^dq}{(2\pi)^d} \sum_{\varepsilon_n>0,\varepsilon_m>0,\varepsilon_k>0,\varepsilon_p>0,\alpha,\beta,\gamma,\sigma,0\le i,l\le 3} \left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}\right)_{il} \Phi_{\{\varepsilon_n,\varepsilon_p\},\{\varepsilon_m,\varepsilon_k\},\{\alpha,\sigma\},\{\beta,\gamma\},i,l} \left(\bar{w}^{\gamma\sigma}_{-\varepsilon_k,\varepsilon_p}\right)_{il} \tag{2.7}$$

where A is defined as follows:

$$A_{\varepsilon_n,\varepsilon_m,i,l} = -\frac{g}{4}\left[q^2 + \frac{1}{D}\left[|\varepsilon_n|\cos(\theta_{\varepsilon_n}) + \Delta\sin(\theta_{\varepsilon_n}) + |\varepsilon_m|\cos(\theta_{\varepsilon_m}) + \Delta\sin(\theta_{\varepsilon_m})\right]\right] +$$
$$+\frac{n_s\alpha}{(1+2\alpha\cos[2\theta_{\varepsilon_m}]+\alpha^2)(1+2\alpha\cos[2\theta_{\varepsilon_n}]+\alpha^2)}\times$$
$$\times\Bigg[\frac{2}{3}(1+\alpha)^2\cos[\theta_{\varepsilon_m}]\cos[\theta_{\varepsilon_n}](1-4\delta_{l0})(1-2\delta_{i3}-2\delta_{i0}) + 2\alpha(\alpha+\cos[2\theta_{\varepsilon_m}])(\alpha+\cos[2\theta_{\varepsilon_n}]) -$$
$$-[\cos[2\theta_{\varepsilon_n}]+\alpha]\left(1+2\alpha\cos[2\theta_{\varepsilon_m}]+\alpha^2\right) - [\cos[2\theta_{\varepsilon_m}]+\alpha]\left(1+2\alpha\cos[2\theta_{\varepsilon_n}]+\alpha^2\right)\Bigg] \tag{2.8}$$

B can be calculated using this expression:

$$B_{\varepsilon_n,\varepsilon_m,i,l} = \frac{2\alpha n_s}{(1+2\alpha\cos[2\theta_{\varepsilon_m}]+\alpha^2)(1+2\alpha\cos[2\theta_{\varepsilon_n}]+\alpha^2)}$$
$$\left[\alpha\sin[2\theta_{\varepsilon_m}]\sin[2\theta_{\varepsilon_n}](1-2\delta_{i2})(1-2\delta_{l0}) - \frac{1}{3}(1-2\delta_{i1})[1+2\delta_{l0}](1-\alpha)^2\sin[\theta_{\varepsilon_m}]\sin[\theta_{\varepsilon_n}]\right] \tag{2.9}$$

And C is this function:

$$C_{\varepsilon_n,\varepsilon_m,i,l} = \frac{\alpha n_s\left[-\frac{8}{3}\sin(\theta_{\varepsilon_n})\sin(\theta_{\varepsilon_m})(1-\alpha)^2\delta_{i2}(1-\delta_{l0}) + 8\alpha\sin[2\theta_{\varepsilon_n}]\sin[2\theta_{\varepsilon_m}]\delta_{i1}\delta_{l0}\right]}{(1+2\alpha\cos[2\theta_{\varepsilon_m}]+\alpha^2)(1+2\alpha\cos[2\theta_{\varepsilon_n}]+\alpha^2)} \tag{2.10}$$

And these three matrices the core of structure of Φ:

$$\Phi_{\{\varepsilon_n,\varepsilon_p\},\{\varepsilon_m,\varepsilon_k\},\{\alpha,\sigma\},\{\beta,\gamma\},i,l} = A_{\varepsilon_n,\varepsilon_m,i,l}\delta_{\alpha\sigma}\delta_{\beta\gamma}\delta_{\varepsilon_n,\varepsilon_P}\delta_{\varepsilon_m,\varepsilon_k} + B_{\varepsilon_n,\varepsilon_m,i,l}\delta_{\alpha\gamma}\delta_{\beta\sigma}\delta_{\varepsilon_n,\varepsilon_k}\delta_{\varepsilon_m,\varepsilon_p} + $$
$$+ C_{\varepsilon_n,\varepsilon_p,i,l}\delta_{\alpha\beta}\delta_{\sigma\gamma}\delta_{\varepsilon_n,\varepsilon_m}\delta_{\varepsilon_k,\varepsilon_p} \tag{2.11}$$

Averaging with quadratic action (2.7) we get standard answer for correlator:

$$\left\langle \left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}(q) \right)_{il} \left(\bar{w}^{\gamma\sigma}_{-\varepsilon_k,\varepsilon_p}(-q) \right)_{il} \right\rangle_{S^{(2)}} = -\frac{1}{2} \left[\Phi^{-1} \right]_{\{\varepsilon_n,\varepsilon_p\},\{\varepsilon_m,\varepsilon_k\},\{\alpha,\sigma\},\{\beta,\gamma\},i,l} \tag{2.12}$$

To find inverse Φ let's assume that Φ^{-1} has the same structure that Φ has:

$$\begin{aligned} \left[\Phi^{-1}\right]_{\{\varepsilon_n,\varepsilon_p\},\{\varepsilon_m,\varepsilon_k\},\{\alpha,\sigma\},\{\beta,\gamma\},i,l} = D_{\varepsilon_n,\varepsilon_m,i,l}\delta_{\alpha\sigma}\delta_{\beta\gamma}\delta_{\varepsilon_n,\varepsilon_P}\delta_{\varepsilon_m,\varepsilon_k} + E_{\varepsilon_n,\varepsilon_m,i,l}\delta_{\alpha\gamma}\delta_{\beta\sigma}\delta_{\varepsilon_n,\varepsilon_k}\delta_{\varepsilon_m,\varepsilon_p} + \\ + F_{\varepsilon_n,\varepsilon_p,i,l}\delta_{\alpha\beta}\delta_{\sigma\gamma}\delta_{\varepsilon_n,\varepsilon_m}\delta_{\varepsilon_k,\varepsilon_p} \end{aligned} \tag{2.13}$$

We need to express the fact that Φ times Φ^{-1} gives the identity matrix:

$$\delta_{\alpha\sigma}\delta_{\beta\gamma}\delta_{\varepsilon_n,\varepsilon_p}\delta_{\varepsilon_m,\varepsilon_k} = \sum_{\varepsilon_u,\varepsilon_v,\eta,\xi} \Phi_{\{\varepsilon_n,\varepsilon_u\},\{\varepsilon_m,\varepsilon_v\},\{\alpha,\eta\},\{\beta,\xi\},i,l} \left[\Phi^{-1}\right]_{\{\varepsilon_u,\varepsilon_p\},\{\varepsilon_v,\varepsilon_k\},\{\eta,\sigma\},\{\xi,\gamma\},i,l} \tag{2.14}$$

This condition leads us to three equations, if they have a solution, then our assumption about the same structure of Φ and Φ^{-1} was correct:

$$\begin{aligned} A_{\varepsilon_n,\varepsilon_m,i,l}D_{\varepsilon_n,\varepsilon_m,i,l} + B_{\varepsilon_n,\varepsilon_m,i,l}E_{\varepsilon_m,\varepsilon_n,i,l} &= 1 \\ A_{\varepsilon_n,\varepsilon_m,i,l}E_{\varepsilon_n,\varepsilon_m,i,l} + B_{\varepsilon_n,\varepsilon_m,i,l}D_{\varepsilon_m,\varepsilon_n,i,l} &= 0 \\ A_{\varepsilon_n,\varepsilon_n,i,l}F_{\varepsilon_n,\varepsilon_p,i,l} + B_{\varepsilon_n,\varepsilon_n,i,l}F_{\varepsilon_n,\varepsilon_p,i,l} + C_{\varepsilon_n,\varepsilon_p,i,l}D_{\varepsilon_p,\varepsilon_p,i,l} + C_{\varepsilon_n,\varepsilon_p,i,l}E_{\varepsilon_p,\varepsilon_p,i,l} &= 0 \end{aligned} \tag{2.15}$$

We can easily solve this system of equations and get the correlator:

$$\begin{aligned} \left\langle \left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}(q) \right)_{il} \left(\bar{w}^{\gamma\sigma}_{-\varepsilon_k,\varepsilon_p}(-q) \right)_{il} \right\rangle_{S^{(2)}} = -\frac{1}{2} \Bigg[\frac{A_{\varepsilon_n,\varepsilon_m,i,l}}{A^2_{\varepsilon_n,\varepsilon_m,i,l} - B^2_{\varepsilon_n,\varepsilon_m,i,l}} \delta_{\alpha\sigma}\delta_{\beta\gamma}\delta_{\varepsilon_n,\varepsilon_P}\delta_{\varepsilon_m,\varepsilon_k} - \\ - \frac{B_{\varepsilon_n,\varepsilon_m,i,l}}{A^2_{\varepsilon_n,\varepsilon_m,i,l} - B^2_{\varepsilon_n,\varepsilon_m,i,l}} \delta_{\alpha\gamma}\delta_{\beta\sigma}\delta_{\varepsilon_n,\varepsilon_k}\delta_{\varepsilon_m,\varepsilon_p} - \\ - \frac{C_{\varepsilon_n,\varepsilon_p,i,l}}{\left(A_{\varepsilon_n,\varepsilon_n,i,l} + B_{\varepsilon_n,\varepsilon_n,i,l}\right)\left(A_{\varepsilon_p,\varepsilon_p,i,l} + B_{\varepsilon_p,\varepsilon_p,i,l}\right)} \delta_{\alpha\beta}\delta_{\sigma\gamma}\delta_{\varepsilon_n,\varepsilon_m}\delta_{\varepsilon_k,\varepsilon_p} \Bigg] \end{aligned} \tag{2.16}$$

Chapter 3

CORRECTION TO THE DOS DUE TO IMPURITY FLUCTUATIONS

To calculate DOS we need to use this linear in Q operator[13]:

$$K_1\left(i\varepsilon_n\right)=\frac{\rho_0}{4}\mathrm{tr}\left\langle\left(\Lambda Q\right)^{\alpha\alpha}_{\varepsilon_n\varepsilon_n}\right\rangle_{S^{(2)}}=\frac{\rho_0}{4}\mathrm{tr}\left(\Lambda R^{-1}\Lambda R\right)^{\alpha\alpha}_{\varepsilon_n\varepsilon_n}-\frac{\rho_0}{8}\mathrm{tr}\left\langle\left(\Lambda R^{-1}\Lambda W^2 R\right)^{\alpha\alpha}_{\varepsilon_n\varepsilon_n}\right\rangle_{S^{(2)}} \tag{3.1}$$

Correction determined by correlators of w:

$$\begin{aligned}\mathrm{tr}\left\langle\left(\Lambda R^{-1}\Lambda W^2 R\right)^{\alpha\alpha}_{\varepsilon_n\varepsilon_n}\right\rangle_{S^{(2)}}&=4\cos^2\left(\theta_{\varepsilon_n}/2\right)\sum_{\beta,\varepsilon_m,i,l}\left\langle\left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}\right)_{il}\left(\bar{w}^{\beta\alpha}_{-\varepsilon_m,\varepsilon_n}\right)_{il}\right\rangle_{S^{(2)}}-\\&-4\sin^2\left(\theta_{\varepsilon_n}/2\right)\sum_{\beta,\varepsilon_m,i,l}\left\langle\left(w^{\beta\alpha}_{\varepsilon_m,-\varepsilon_n}\right)_{il}\left(\bar{w}^{\alpha\beta}_{-\varepsilon_n,\varepsilon_m}\right)_{il}\right\rangle_{S^{(2)}}=\\=2\int\frac{d^2q}{(2\pi)^2}&\left(\cos\left(\theta_{\varepsilon_n}\right)\sum_{i,l}\frac{B_{\varepsilon_n,\varepsilon_n,i,l}}{A^2_{\varepsilon_n,\varepsilon_n,i,l}-B^2_{\varepsilon_n,\varepsilon_n,i,l}}+\cos\left(\theta_{\varepsilon_n}\right)\sum_{i,l}\frac{C_{\varepsilon_n,\varepsilon_n,i,l}}{\left(A_{\varepsilon_n,\varepsilon_n,i,l}+B_{\varepsilon_n,\varepsilon_n,i,l}\right)^2}\right)\end{aligned} \tag{3.2}$$

So we get DOS with $1/g$ correction:

$$K_1\left(i\varepsilon_n\right)=\rho_0\cos\left(\theta_{\varepsilon_n}\right)\left(1-\frac{1}{4}\int\frac{d^2q}{(2\pi)^2}\sum_{i,l}\left[\frac{B_{\varepsilon_n,\varepsilon_n,i,l}}{A^2_{\varepsilon_n,\varepsilon_n,i,l}-B^2_{\varepsilon_n,\varepsilon_n,i,l}}+\frac{C_{\varepsilon_n,\varepsilon_n,i,l}}{\left(A_{\varepsilon_n,\varepsilon_n,i,l}+B_{\varepsilon_n,\varepsilon_n,i,l}\right)^2}\right]\right) \tag{3.3}$$

It can be rewrite using symmetries of matrices:

$$\begin{aligned}\left\langle\rho\left(E,r\right)\right\rangle=\rho_0\cos\left(\theta_{\varepsilon_n}\right)&\left[1-\frac{1}{4}\int\frac{d^2q}{(2\pi)^2}\left[\frac{2B_{\varepsilon_n,\varepsilon_n,0,0}}{A^2_{\varepsilon_n,\varepsilon_n,0,0}-B^2_{\varepsilon_n,\varepsilon_n,0,0}}+\frac{6B_{\varepsilon_n,\varepsilon_n,1,0}}{A^2_{\varepsilon_n,\varepsilon_n,1,0}-B^2_{\varepsilon_n,\varepsilon_n,1,0}}+\right.\right.\\&\left.\left.+\frac{C_{\varepsilon_n,\varepsilon_n,1,0}}{\left(A_{\varepsilon_n,\varepsilon_n,1,0}+B_{\varepsilon_n,\varepsilon_n,1,0}\right)^2}+\frac{3C_{\varepsilon_n,\varepsilon_n,2,1}}{\left(A_{\varepsilon_n,\varepsilon_n,2,1}+B_{\varepsilon_n,\varepsilon_n,2,1}\right)^2}\right]\right]\end{aligned} \tag{3.4}$$

3.1 Correction to DOS close to the edge of the spectrum

Let's define ψ, and function $F(\psi)$ using $\mu = \frac{2\alpha}{1+\alpha^2}, \eta = \frac{n_s \mu}{4 Z_\omega \Delta}$:

$$\theta_{\varepsilon_n} = \frac{\pi}{2} + i\psi,\ F(\psi) = -\frac{E}{\Delta}\cosh\psi + \sinh\psi - \frac{\eta \sinh 2\psi}{2\left(1-\mu\cosh 2\psi\right)} \tag{3.5}$$

If we assume $i\varepsilon = E + i0$, then Usadel equation (1.17) becomes:

$$F(\psi) = 0 \tag{3.6}$$

$\frac{dF(\psi)}{d\psi}$ is the small enough close to the edge of spectrum because $\frac{dF(\psi_g)}{d\psi} = 0$:

$$\frac{dF(\psi)}{d\psi} = -\frac{E}{\Delta}\sinh\psi + \cosh\psi + \eta\frac{\mu - \cosh 2\psi}{\left(1-\mu\cosh 2\psi\right)^2} \tag{3.7}$$

And one of the terms in correction to DOS determined by this small derivative:

$$\begin{aligned} \int \frac{d^2q}{(2\pi)^2}\frac{C_{\varepsilon_n,\varepsilon_n,1,0}}{\left(A_{\varepsilon_n,\varepsilon_n,1,0}+B_{\varepsilon_n,\varepsilon_n,1,0}\right)^2} &= \\ = \frac{1}{\pi g}\frac{\frac{8\alpha^2 n_s \sin^2[2\theta_{\varepsilon_n}]}{(1+2\alpha\cos[2\theta_{\varepsilon_n}]+\alpha^2)^2}}{\frac{g}{4L^2}+\frac{g}{2D}\left[|\varepsilon_n|\cos\left(\theta_{\varepsilon_n}\right)+\Delta\sin\left(\theta_{\varepsilon_n}\right)\right]+\frac{4n_s\alpha((1+\alpha^2)\cos[2\theta_{\varepsilon_n}]+2\alpha)}{(1+2\alpha\cos[2\theta_{\varepsilon_n}]+\alpha^2)^2}} &= \\ = \frac{1}{\pi g}\frac{\frac{2\mu^2 n_s \sinh^2[2\psi]}{(1-\mu\cosh[2\psi])^2}}{\frac{g}{4L^2}+\frac{g\Delta}{2D}\left[-\frac{E}{\Delta}\sinh\left(\psi\right)+\cosh\left[\psi\right]\right]+\frac{2n_s\mu(-\cosh[2\psi]+\mu)}{(1-\mu\cosh[2\psi])^2}} &\approx \\ \approx -\frac{1}{\pi g}\frac{\eta\mu\sinh^2\left[2\psi\right]}{\left(1-\mu\cosh\left[2\psi\right]\right)^2\frac{dF(\psi)}{d\psi}} & \end{aligned} \tag{3.8}$$

Let's find $\psi - \psi_g$ when $0 < E - E_g \ll 1$

$$\begin{aligned} 0 = F(\psi, E) - F(\psi_g.E_g) = -\frac{E}{\Delta}\cosh\psi + \sinh\psi - \frac{\eta\sinh 2\psi}{2\left(1-\mu\cosh 2\psi\right)} = \\ = F'(\psi_g)\left(\psi-\psi_g\right) + \frac{d}{dE}F(\psi_g)\left(E-E_g\right) + \frac{1}{2}F''(\psi_g)\left(\psi-\psi_g\right)^2 + \frac{1}{2}\frac{d^2}{dE^2}F(\psi_g)\left(E-E_g\right)^2 \end{aligned} \tag{3.9}$$

$F'(\psi_g) = 0$ by the determination of ψ_g, and second order of $E - E_g$ is excess of accuracy, according to this we can get:

$$(\psi-\psi_g)=\sqrt{\frac{-2\frac{d}{dE}F(\psi_g)}{F''(\psi_g)}\left(E-E_g\right)}=\sqrt{\frac{2\cosh\psi_g}{F''(\psi_g)}\frac{(E-E_g)}{\Delta}} \tag{3.10}$$

and now we get the order of smallness of $\frac{dF(\psi)}{d\psi}$

$$\frac{dF(\psi)}{d\psi}=F''(\psi_g)\left(\psi-\psi_g\right)=\sqrt{2F''(\psi_g)\cosh\psi_g\frac{(E-E_g)}{\Delta}} \tag{3.11}$$

This is equation that determines ψ_g:

$$\frac{1+\mu\left(\cosh 2\psi_g-2\right)}{\left(1-\mu\cosh 2\psi_g\right)^2}\cosh^3\psi_g=\frac{1}{\eta} \tag{3.12}$$

Second order derivative is negative:

$$\frac{d^2F(\psi)}{d\psi^2}=-\frac{\eta\sinh[2\psi](6-17\mu^2+12\mu\cosh[2\psi]-\mu^2\cosh[4\psi])}{4(1-\mu\cosh[2\psi])^3}<0 \tag{3.13}$$

So we get imaginary correction:

$$\int\frac{d^2q}{(2\pi)^2}\frac{C_{\varepsilon_n,\varepsilon_n,1,0}}{\left(A_{\varepsilon_n,\varepsilon_n,1,0}+B_{\varepsilon_n,\varepsilon_n,1,0}\right)^2}\approx\frac{-i}{\pi g}\frac{\eta\mu\sinh^2\left[2\psi_g\right]}{\left(1-\mu\cosh\left[2\psi_g\right]\right)^2\sqrt{2|F''(\psi_g)|\cosh\psi_g\frac{(E-E_g)}{\Delta}}} \tag{3.14}$$

$$\langle\delta\rho\left(E,r\right)\rangle=\frac{\rho_0\eta\mu}{4\pi g}\frac{\sinh\left[\psi_g\right]\sinh^2\left[2\psi_g\right]}{\left(1-\mu\cosh\left[2\psi_g\right]\right)^2\sqrt{2\left|F''(\psi_g)\right|\cosh\psi_g\frac{(E-E_g)}{\Delta}}} \tag{3.15}$$

The bare DOS without correction close to E_g looks like:

$$\langle\rho_0\left(E,r\right)\rangle=\rho_0\cosh\psi_g\sqrt{\frac{2\cosh\psi_g}{|F''(\psi_g)|}\frac{(E-E_g)}{\Delta}} \tag{3.16}$$

We must say that correction to DOS can't be larger then bare DOS, so we can't consider very close to edge of spectrum:

$$\frac{(E-E_g)}{\Delta}\gg\frac{\eta\mu}{2\pi g}\frac{\sinh^3\left[\psi_g\right]\cosh^2\left[\psi_g\right]}{\left(1-\mu\cosh\left[2\psi_g\right]\right)^2} \tag{3.17}$$

3.1.1 Correction to DOS close to the edge of the spectrum in 3D

If we still consider $\frac{D}{2\Delta l^2} \gg \frac{dF(\psi)}{d\psi} \gg \frac{D}{2\Delta L^2}$:

$$
\begin{aligned}
\int \frac{d^3q}{(2\pi)^3} \frac{C_{\varepsilon_n,\varepsilon_n,1,0}}{\left(A_{\varepsilon_n,\varepsilon_n,1,0} + B_{\varepsilon_n,\varepsilon_n,1,0}\right)^2} &= \frac{8}{\pi^2 g^2} \frac{2\mu^2 n_s \sinh^2[2\psi]}{(1-\mu\cosh[2\psi])^2} \int\limits_{1/L}^{1/l} dq \frac{q^2}{\left(\frac{2\Delta}{D}\frac{dF(\psi)}{d\psi} + q^2\right)^2} = \\
= \frac{8}{\pi^2 g^2} \frac{\mu^2 n_s \sinh^2[2\psi]}{(1-\mu\cosh[2\psi])^2} &\left(\frac{1}{\sqrt{\frac{2\Delta}{D}\frac{dF(\psi)}{d\psi}}} \arctan \frac{q}{\sqrt{\frac{2\Delta}{D}\frac{dF(\psi)}{d\psi}}} - \frac{q}{\frac{2\Delta}{D}\frac{dF(\psi)}{d\psi} + q^2} \right)\Bigg|_{1/L}^{1/l} = \\
= \frac{4}{\pi g^2} \frac{\mu^2 n_s \sinh^2[2\psi]}{(1-\mu\cosh[2\psi])^2} \frac{1}{\sqrt{\frac{2\Delta}{D}\frac{dF(\psi)}{d\psi}}} &= \frac{4}{\pi g^2} \frac{\mu^2 n_s \sinh^2[2\psi]}{(1-\mu\cosh[2\psi])^2} \frac{1}{\sqrt{\frac{2\Delta}{D}\frac{dF(\psi)}{d\psi}}} = \\
&= \frac{4}{\pi g^2} \frac{\mu^2 n_s \sinh^2[2\psi]}{(1-\mu\cosh[2\psi])^2} \frac{1}{\sqrt{\frac{2\Delta}{D}} \left(2F''(\psi_g)\cosh\psi_g \frac{E-E_g}{\Delta}\right)^{1/4}}
\end{aligned}
\tag{3.18}
$$

Then we get sharp edge of the spectrum:

$$
\langle \delta\rho(E,r) \rangle = \frac{\rho_0 \mu^2 n_s}{2\pi g^2} \frac{\sinh[\psi]\sinh^2[2\psi]}{(1-\mu\cosh[2\psi])^2} \frac{1}{\sqrt{\frac{\Delta}{D}} \left(2\left|F''(\psi_g)\right|\cosh\psi_g \frac{E-E_g}{\Delta}\right)^{1/4}} \tag{3.19}
$$

3.1.2 Abrikosov-Gor'kov approximation($\alpha \ll 1$)

So our condition (3.17) will be simplified:

$$
\frac{(E-E_g)}{\Delta} \gg \frac{\mu}{2\pi g \eta^{2/3}} \tag{3.20}
$$

We see that this correction is negligible in Bohr regime:

$$
\langle \delta\rho(E,r) \rangle = \frac{\rho_0 \mu}{\pi g \eta^{2/3}} \frac{1}{\sqrt{6\frac{(E-E_g)}{\Delta}}}, \quad \frac{(E-E_g)}{\Delta} \gg \frac{\mu}{2\pi g \eta^{2/3}} \tag{3.21}
$$

Chapter 4

FINDING FLUCTUATIONS OF LDOS

One part of $P_2^{\alpha_1\alpha_2}(i\varepsilon_n, i\varepsilon_m)$ can be written to the second order in W

$$\begin{aligned}\langle\langle \mathrm{tr}Q_{nn}^{\alpha_1\alpha_1}(r)\,\mathrm{tr}Q_{mm}^{\alpha_2\alpha_2}(r)\rangle\rangle &= \langle \mathrm{tr}Q_{nn}^{\alpha_1\alpha_1}(r)\,\mathrm{tr}Q_{mm}^{\alpha_2\alpha_2}(r)\rangle - \langle \mathrm{tr}Q_{nn}^{\alpha_1\alpha_1}(r)\rangle\langle \mathrm{tr}Q_{mm}^{\alpha_2\alpha_2}(r)\rangle = \\ &= \left\langle \mathrm{tr}\left[R^{-1}WR\right]_{nn}^{\alpha_1\alpha_1}(r)\,\mathrm{tr}\left[R^{-1}WR\right]_{mm}^{\alpha_2\alpha_2}(r)\right\rangle = \\ &= 16\mathrm{sgn}\varepsilon_n\mathrm{sgn}\varepsilon_m \sin(\theta_{\varepsilon_n})\sin(\theta_{\varepsilon_m})\left\langle\left(w_{\varepsilon_n,-\varepsilon_n}^{\alpha_1\alpha_1}\right)_{10}\left(\bar{w}_{-\varepsilon_m,\varepsilon_m}^{\alpha_2\alpha_2}\right)_{10}\right\rangle = \\ &= \int\frac{d^dq}{(2\pi)^d}\frac{8\mathrm{sgn}\varepsilon_n\mathrm{sgn}\varepsilon_m\sin(\theta_{\varepsilon_n})\sin(\theta_{\varepsilon_m})\,C_{\varepsilon_n,\varepsilon_m,1,0}}{(A_{\varepsilon_n,\varepsilon_n,1,0}+B_{\varepsilon_n,\varepsilon_n,1,0})(A_{\varepsilon_m,\varepsilon_m,1,0}+B_{\varepsilon_m,\varepsilon_m,1,0})}\end{aligned} \tag{4.1}$$

Another part can be written to the second order in W too:

$$\langle \mathrm{tr}Q_{nm}^{\alpha_1\alpha_2}(r)\,Q_{mn}^{\alpha_2\alpha_1}(r)\rangle = \left\langle \mathrm{tr}\left[R^{-1}WR\right]_{nm}^{\alpha_1\alpha_2}(r)\left[R^{-1}WR\right]_{mn}^{\alpha_2\alpha_1}(r)\right\rangle \tag{4.2}$$

We can easily calculate this term:

$$\begin{aligned}&\left\langle \mathrm{tr}\left[R^{-1}WR\right]_{\varepsilon_n,\varepsilon_m}^{\alpha_1\alpha_2}(r)\left[R^{-1}WR\right]_{\varepsilon_m,\varepsilon_n}^{\alpha_2\alpha_1}(r)\right\rangle_{i\varepsilon_n=E+i0^+,i\varepsilon_m=E'+i0^+} = \\ &= 4\sum_{i,l}\Bigg[\cos^2(\theta_{\varepsilon_n}/2)\sin^2(\theta_{\varepsilon_m}/2)\left\langle\left(w_{\varepsilon_n,-\varepsilon_m}^{\alpha_1\alpha_2}\right)_{il}\left(\bar{w}_{-\varepsilon_m,\varepsilon_n}^{\alpha_2\alpha_1}\right)_{il}\right\rangle + \\ &+\sin^2(\theta_{\varepsilon_n}/2)\cos^2(\theta_{\varepsilon_m}/2)\left\langle\left(\bar{w}_{-\varepsilon_n,\varepsilon_m}^{\alpha_1\alpha_2}\right)_{il}\left(w_{\varepsilon_m,-\varepsilon_n}^{\alpha_2\alpha_1}\right)_{il}\right\rangle - \\ &-\frac{1}{4}(1-2\delta_{l0})(1-2\delta_{i2})\sin(\theta_{\varepsilon_n})\sin(\theta_{\varepsilon_m})\times \\ &\times\left(\left\langle\left(w_{\varepsilon_n,-\varepsilon_m}^{\alpha_1\alpha_2}\right)_{il}\left(\bar{w}_{-\varepsilon_n,\varepsilon_m}^{\alpha_1\alpha_2}\right)_{il}\right\rangle + \left\langle\left(w_{\varepsilon_m,-\varepsilon_n}^{\alpha_2\alpha_1}\right)_{il}\left(\bar{w}_{-\varepsilon_m,\varepsilon_n}^{\alpha_2\alpha_1}\right)_{il}\right\rangle\right)\Bigg] = \\ &= \int\frac{d^dq}{(2\pi)^d}\sum_{i,l}\Bigg[-2\cos^2(\theta_{\varepsilon_n}/2)\sin^2(\theta_{\varepsilon_m}/2)\frac{A_{\varepsilon_n,\varepsilon_m,i,l}}{A_{\varepsilon_n,\varepsilon_m,i,l}^2-B_{\varepsilon_n,\varepsilon_m,i,l}^2} - \\ &-2\sin^2(\theta_{\varepsilon_n}/2)\cos^2(\theta_{\varepsilon_m}/2)\frac{A_{-\varepsilon_n,-\varepsilon_m,i,l}}{A_{-\varepsilon_n,-\varepsilon_m,i,l}^2-B_{-\varepsilon_n,-\varepsilon_m,i,l}^2} - \\ &-\frac{1}{2}(1-2\delta_{l0})(1-2\delta_{i2})\sin(\theta_{\varepsilon_n})\sin(\theta_{\varepsilon_m})\left(\frac{B_{\varepsilon_n,\varepsilon_m,i,l}}{A_{\varepsilon_n,\varepsilon_m,i,l}^2-B_{\varepsilon_n,\varepsilon_m,i,l}^2}+\frac{B_{-\varepsilon_n,-\varepsilon_m,i,l}}{A_{-\varepsilon_n,-\varepsilon_m,i,l}^2-B_{-\varepsilon_n,-\varepsilon_m,i,l}^2}\right)\Bigg]\end{aligned} \tag{4.3}$$

Summaring two contributions we obtain Retarded-Retarded(RR) correlation function $P_2^{\alpha_1\alpha_2}(i\varepsilon_n, i\varepsilon_m)$:

$$P_2^{\alpha_1\alpha_2}(i\varepsilon_n, i\varepsilon_m)_{i\varepsilon_n=E+i0^+, i\varepsilon_m=E'+i0^+} = \langle\langle \mathrm{tr}Q_{nn}^{\alpha_1\alpha_1}(r)\,\mathrm{tr}Q_{mm}^{\alpha_2\alpha_2}(r)\rangle\rangle - 2\langle \mathrm{tr}Q_{nm}^{\alpha_1\alpha_2}(r)\,Q_{mn}^{\alpha_2\alpha_1}(r)\rangle =$$
$$= \int \frac{d^dq}{(2\pi)^d}\left\{\frac{8\sin(\theta_{\varepsilon_n})\sin(\theta_{\varepsilon_m})C_{\varepsilon_n,\varepsilon_m,1,0}}{(A_{\varepsilon_n,\varepsilon_n,1,0}+B_{\varepsilon_n,\varepsilon_n,1,0})(A_{\varepsilon_m,\varepsilon_m,1,0}+B_{\varepsilon_m,\varepsilon_m,1,0})}+\right.$$
$$+\sum_{i,l}\frac{4\left[\cos^2(\theta_{\varepsilon_n}/2)\sin^2(\theta_{\varepsilon_m}/2)+\sin^2(\theta_{\varepsilon_n}/2)\cos^2(\theta_{\varepsilon_m}/2)\right]A_{\varepsilon_n,\varepsilon_m,i,l}+}{A^2_{\varepsilon_n,\varepsilon_m,i,l}-B^2_{\varepsilon_n,\varepsilon_m,i,l}}+$$
$$\left.+\sum_{i,l}\frac{2(1-2\delta_{l0})(1-2\delta_{i2})\sin(\theta_{\varepsilon_n})\sin(\theta_{\varepsilon_m})B_{\varepsilon_n,\varepsilon_m,i,l}}{A^2_{\varepsilon_n,\varepsilon_m,i,l}-B^2_{\varepsilon_n,\varepsilon_m,i,l}}\right\} \tag{4.4}$$

It can be simplify for case when $E = E'$:

$$P_2^{\alpha_1\alpha_2}(i\varepsilon_n, i\varepsilon_n)_{i\varepsilon_n=E+i0^+} =$$
$$= \int \frac{d^dq}{(2\pi)^d}\left\{\frac{8\sin^2(\theta_{\varepsilon_n})C_{\varepsilon_n,\varepsilon_n,1,0}}{(A_{\varepsilon_n,\varepsilon_n,1,0}+B_{\varepsilon_n,\varepsilon_n,1,0})^2}+2\sin^2(\theta_{\varepsilon_n})\sum_{i,l}\frac{1}{A_{\varepsilon_n,\varepsilon_n,i,l}-(1-2\delta_{l0})(1-2\delta_{i2})B_{\varepsilon_n,\varepsilon_n,i,l}}\right\} \tag{4.5}$$

Similarly we can get Retarded-Advanced(RA) correlation function:

$$\mathrm{Re}P_2^{\alpha_1\alpha_2}(i\varepsilon_n, i\varepsilon_m)_{i\varepsilon_k=E+i0^+, i\varepsilon_p=E'-i0^+} =$$
$$= \mathrm{Re}\int \frac{d^dq}{(2\pi)^d}\left\{-\frac{8\sin(\theta_{\varepsilon_k})\sin(\theta_{\varepsilon_p})C_{\varepsilon_k,\varepsilon_p,1,0}}{(A_{\varepsilon_k,\varepsilon_k,1,0}+B_{\varepsilon_k,\varepsilon_k,1,0})(A_{\varepsilon_p,\varepsilon_p,1,0}+B_{\varepsilon_p,\varepsilon_p,1,0})}+\right.$$
$$+\sum_{i,l}\left[2\left(1+\cos(\theta_{\varepsilon_k})\cos(\theta_{\varepsilon_p})\right)\frac{A_{\varepsilon_k,-\varepsilon_p,i,l}}{A^2_{\varepsilon_k,-\varepsilon_p,i,l}-B^2_{\varepsilon_k,-\varepsilon_p,i,l}}-\right.$$
$$\left.\left.-2(1-2\delta_{l0})(1-2\delta_{i2})\sin(\theta_{\varepsilon_k})\sin(\theta_{\varepsilon_p})\frac{B_{\varepsilon_k,-\varepsilon_p,i,l}}{A^2_{\varepsilon_k,-\varepsilon_p,i,l}-B^2_{\varepsilon_k,-\varepsilon_p,i,l}}\right]\right\} \tag{4.6}$$

4.1 Fluctuations of LDOS below the energy gap

Let's simplify RA using that DOS is equal to zero, so $\cos[\theta_{\varepsilon_k}]$ is fully imaginary, so for $i\varepsilon_k = E+i0^+, i\varepsilon_p = E'-i0^+$ when $\theta_{\varepsilon_k} = \bar{\theta}_{\varepsilon_p}$ because of Usadel equation (1.17) we get this:

$$\cos[\theta_{\varepsilon_k}] = -\cos[\theta_{\varepsilon_p}],\ \sin[\theta_{\varepsilon_k}] = \sin[\theta_{\varepsilon_p}] \tag{4.7}$$

So RA correlation function can be simplified using this relations:

$$
\begin{aligned}
&\mathrm{Re}P_2^{\alpha_1\alpha_2}\left(i\varepsilon_n, i\varepsilon_m\right)_{i\varepsilon_k=E+i0^+, i\varepsilon_p=E-i0^+} = \\
&= \mathrm{Re}\int \frac{d^d q}{(2\pi)^d}\Bigg\{-\frac{8\sin\left(\theta_{\varepsilon_k}\right)\sin\left(\theta_{\varepsilon_p}\right)C_{\varepsilon_k,\varepsilon_p,1,0}}{\left(A_{\varepsilon_k,\varepsilon_k,1,0}+B_{\varepsilon_k,\varepsilon_k,1,0}\right)\left(A_{\varepsilon_p,\varepsilon_p,1,0}+B_{\varepsilon_p,\varepsilon_p,1,0}\right)}+ \\
&+2\sin^2\left(\theta_{\varepsilon_k}\right)\sum_{i,l}\left[\frac{A_{\varepsilon_k,-\varepsilon_p,i,l}-\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}{A^2_{\varepsilon_k,-\varepsilon_p,i,l}-B^2_{\varepsilon_k,-\varepsilon_p,i,l}}\right]\Bigg\} = \\
&= \mathrm{Re}\int \frac{d^d q}{(2\pi)^d}\Bigg\{-\frac{8\sin\left(\theta_{\varepsilon_k}\right)\sin\left(\theta_{\varepsilon_p}\right)C_{\varepsilon_k,\varepsilon_p,1,0}}{\left(A_{\varepsilon_k,\varepsilon_k,1,0}+B_{\varepsilon_k,\varepsilon_k,1,0}\right)\left(A_{\varepsilon_p,\varepsilon_p,1,0}+B_{\varepsilon_p,\varepsilon_p,1,0}\right)}+ \\
&+2\sin^2\left(\theta_{\varepsilon_k}\right)\sum_{i,l}\frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l}+\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}\Bigg\}
\end{aligned} \tag{4.8}
$$

Terms with C in RA function and RR function reduced with each other:

$$
C_{\varepsilon_k,\varepsilon_p,1,0} = -C_{\varepsilon_n,\varepsilon_n,1,0} \tag{4.9}
$$

$$
\frac{8\sin^2\left(\theta_{\varepsilon_n}\right)C_{\varepsilon_n,\varepsilon_n,1,0}}{\left(A_{\varepsilon_n,\varepsilon_n,1,0}+B_{\varepsilon_n,\varepsilon_n,1,0}\right)^2}+\frac{8\sin^2\left(\theta_{\varepsilon_k}\right)C_{\varepsilon_k,\varepsilon_p,1,0}}{\left(A_{\varepsilon_k,\varepsilon_k,1,0}+B_{\varepsilon_k,\varepsilon_k,1,0}\right)\left(A_{\varepsilon_p,\varepsilon_p,1,0}+B_{\varepsilon_p,\varepsilon_p,1,0}\right)} = 0 \tag{4.10}
$$

Denominators in RA and RR functions can be rewritten using $S_{i,l}$ and $S'_{i,l}$:

$$
A_{\varepsilon_k,\varepsilon_k,i,l}-\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,\varepsilon_k,i,l} = -\frac{g}{4}q^2-\frac{g}{4}S_{i,l} \tag{4.11}
$$

$$
A_{\varepsilon_k,-\varepsilon_p,i,l}+\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l} = -\frac{g}{4}q^2-\frac{g}{4}S'_{i,l} \tag{4.12}
$$

$$
\begin{aligned}
S_{i,l} = \frac{1}{D}\left[2\left|\varepsilon_k\right|\cos\left(\theta_{\varepsilon_k}\right)+2\Delta\sin\left(\theta_{\varepsilon_k}\right)\right]-\frac{8n_s\alpha\left(\left(1+\alpha^2\right)\cos\left[2\theta_{\varepsilon_k}\right]+2\alpha\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\times \\
\times\left[\frac{1}{3}\left(1-4\delta_{l0}\right)\left(1-2\delta_{i3}-2\delta_{i0}\right)-1\right]
\end{aligned} \tag{4.13}
$$

$$S'_{i,l} = \frac{1}{D}\left[2\left|\varepsilon_k\right|\cos\left(\theta_{\varepsilon_k}\right) + 2\Delta\sin\left(\theta_{\varepsilon_k}\right)\right] - \frac{8n_s\alpha\left(\left(1+\alpha^2\right)\cos\left[2\theta_{\varepsilon_k}\right] + 2\alpha\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\times$$
$$\times\left[-\frac{1}{3}\left(1-4\delta_{l0}\right)\left(1-2\delta_{i3}-2\delta_{i0}\right)-1\right] \quad (4.14)$$

We see that there are part that doesn't depend on indices i and l and part that depend on:

$$S_{i,l} = S_0 + I_{il},\ S'_{i,l} = S_0 - I_{il} \quad (4.15)$$

After integration we get logarithmic expression:

$$\int \frac{d^2q}{\left(2\pi\right)^2}\sin^2\left(\theta_{\varepsilon_k}\right)\sum_{i,l}\left[\frac{1}{A_{\varepsilon_k,\varepsilon_k,i,l} - \left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,\varepsilon_k,i,l}} - \right.$$
$$\left. - \frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l} + \left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}\right] =$$
$$= -\frac{4}{g}\sum_{i,l}\sin^2\left(\theta_{\varepsilon_k}\right)\int\frac{qdq}{\left(2\pi\right)}\left[\frac{1}{q^2+S_{i,l}} - \frac{1}{q^2+S'_{i,l}}\right] = -\frac{1}{\pi g}\sum_{i,l}\sin^2\left(\theta_{\varepsilon_k}\right)\ln\left[\frac{q^2+S_{i,l}}{q^2+S'_{i,l}}\right]\Bigg|_{1/L}^{1/l} \quad (4.16)$$

It can be rewritten using S_0 and I_{il}:

$$K_2\left(E,E',r\right) = -\frac{\rho_0^2}{32\pi g}\sin^2\left(\theta_{\varepsilon_k}\right)\sum_{i,l}\ln\frac{S'_{i,l}+\frac{1}{L^2}}{S_{i,l}+\frac{1}{L^2}}\frac{l^2S_{i,l}+1}{l^2S'_{i,l}+1} =$$
$$= -\frac{\rho_0^2}{32\pi g}\sin^2\left(\theta_{\varepsilon_k}\right)\sum_{i,l}\ln\frac{S_0+\frac{1}{L^2}-I_{il}}{S_0+\frac{1}{L^2}+I_{il}}\frac{1+l^2S_0-l^2I_{il}}{1+l^2S_0+l^2I_{il}} \quad (4.17)$$

We see that odd orders of I_{il} expansion is equal to zero after summation by i and l:

$$\sum_{i,l}\left(I_{il}\right)^{2n+1} \sim \sum_{l}\left(1-4\delta_{l0}\right)^{2n+1}\sum_{i}\left(1-2\delta_{i3}-2\delta_{i0}\right) = 0 \quad (4.18)$$

We see that function is odd in I_{il} if we make Teylor expansion by I_{il} it will contain only odd orders, and as we show summation by odd orders get zero, so when DOS is equal to zero fluctuations equal to zero too:

$$K_2\left(E,E',r\right) = 0 \quad (4.19)$$

4.2 Fluctuations of LDOS above the energy gap

Analogous when DOS is equal to zero we can write the fact that $\cos\left[\theta_{\varepsilon_k}\right]$ is real and $\sin\left[\theta_{\varepsilon_k}\right]$ is imaginary:

$$\cos\left[\theta_{\varepsilon_k}\right] = \cos\left[\theta_{\varepsilon_p}\right],\ \sin\left[\theta_{\varepsilon_k}\right] = -\sin\left[\theta_{\varepsilon_p}\right] \tag{4.20}$$

So our RA correlation function could be simplified as:

$$\begin{gathered} \mathrm{Re}P_2^{\alpha_1\alpha_2}\left(i\varepsilon_n, i\varepsilon_m\right)_{i\varepsilon_k=E+i0^+, i\varepsilon_p=E-i0^+} = \\ = \mathrm{Re}\int \frac{d^dq}{(2\pi)^d}\left\{-\frac{8\sin\left(\theta_{\varepsilon_k}\right)\sin\left(\theta_{\varepsilon_p}\right)C_{\varepsilon_k,\varepsilon_p,1,0}}{\left(A_{\varepsilon_k,\varepsilon_k,1,0}+B_{\varepsilon_k,\varepsilon_k,1,0}\right)\left(A_{\varepsilon_p,\varepsilon_p,1,0}+B_{\varepsilon_p,\varepsilon_p,1,0}\right)}+\right. \\ +2\sum_{i,l}\left[\frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l}-\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}+\right. \\ \left.\left. +\cos^2\left(\theta_{\varepsilon_k}\right)\frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l}+\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}\right]\right\} \end{gathered} \tag{4.21}$$

In the expression above the denominators can be written as follows:

$$A_{\varepsilon_k,-\varepsilon_p,i,l}+\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l} = -\frac{g}{4}q^2-\frac{g}{4}S_{ij} \tag{4.22}$$

$$A_{\varepsilon_k,-\varepsilon_p,i,l}-\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l} = -\frac{g}{4}q^2-\frac{g}{4}S_0-\frac{g}{4}S'_{ij} \tag{4.23}$$

where

$$S_0 = \frac{8n_s\alpha^2}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right) \tag{4.24}$$

$$S'_{ij} = -\frac{8n_s\alpha}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)}\left[\frac{1}{3}\left(1-4\delta_{l0}\right)\left(1-2\delta_{i3}-2\delta_{i0}\right)-1\right] \tag{4.25}$$

$$S_{ij}=-\frac{8n_s\alpha\left(\left(1+\alpha^2\right)\cos\left[2\theta_{\varepsilon_k}\right]+2\alpha\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\left[\frac{1}{3}\left(1-4\delta_{l0}\right)\left(1-2\delta_{i3}-2\delta_{i0}\right)-1\right] \tag{4.26}$$

When $1/L^2 \ll S_0+S'_{ij} \ll 1/l^2$ we can integrate

$$\begin{aligned}
\mathrm{Re}\int\frac{d^2q}{(2\pi)^2}\sum_{i,l}\frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l}-\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}=\\
=-\frac{1}{\pi g}\sum_{i,l}\int dq^2\frac{1}{q^2+S_0+S'_{ij}}=-\frac{1}{\pi g}\sum_{i,l}\ln\left(q^2+S_0+S'_{ij}\right)\Bigg|_{1/L}^{1/l}=\\
=-\frac{1}{\pi g}\sum_{i,l}\ln\left(\frac{1/l^2}{S_0+S'_{ij}}\right)=\frac{2}{\pi g}\Bigg[\ln\left(\frac{8n_s\alpha^2l^2}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)\right)+\\
+\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)+2\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)+\\
+3\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)+4/3\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)+\\
+3\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)+2/3\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)\Bigg]
\end{aligned} \tag{4.27}$$

For other fraction we get logarithmically large by $\frac{L}{l}$ answer:

$$\begin{aligned}
\mathrm{Re}\int\frac{d^2q}{(2\pi)^2}\sum_{i,l}\frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l}+\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}=\\
=-\frac{1}{\pi g}\sum_{i,l}\int dq^2\frac{1}{q^2+S_{ij}}=-\frac{1}{\pi g}\sum_{i,l}\ln\left(q^2+S_{ij}\right)\Bigg|_{1/L}^{1/l}=\\
\approx-\frac{2}{\pi g}\left[2\ln\left(\frac{L}{l}\right)-7\ln\left(\frac{l^2n_s\alpha\left(\left(1+\alpha^2\right)\cos\left[2\theta_{\varepsilon_k}\right]+2\alpha\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)\right]
\end{aligned} \tag{4.28}$$

So this is RA correlation function:

$$
\begin{aligned}
&\mathrm{Re}P_2^{\alpha_1\alpha_2}\left(i\varepsilon_n, i\varepsilon_m\right)_{i\varepsilon_k=E+i0^+, i\varepsilon_p=E-i0^+} = \\
&= \mathrm{Re}\int \frac{d^d q}{(2\pi)^d}\left\{-\frac{8\sin\left(\theta_{\varepsilon_k}\right)\sin\left(\theta_{\varepsilon_p}\right)C_{\varepsilon_k,\varepsilon_p,1,0}}{\left(A_{\varepsilon_k,\varepsilon_k,1,0}+B_{\varepsilon_k,\varepsilon_k,1,0}\right)\left(A_{\varepsilon_p,\varepsilon_p,1,0}+B_{\varepsilon_p,\varepsilon_p,1,0}\right)}+\right. \\
&+2\mathrm{Re}\int \frac{d^d q}{(2\pi)^d}\left\{\sum_{i,l}\left[\frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l}-\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}+\right.\right. \\
&\left.\left.+\cos^2\left(\theta_{\varepsilon_k}\right)\frac{1}{A_{\varepsilon_k,-\varepsilon_p,i,l}+\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,-\varepsilon_p,i,l}}\right]\right\} \approx \\
&\approx -\frac{4}{\pi g}\left[\cos^2\left(\theta_{\varepsilon_k}\right)\left\{2\ln\left(\frac{L}{l}\right)-7\ln\left(\frac{l^2 n_s\alpha\left(\left(1+\alpha^2\right)\cos\left[2\theta_{\varepsilon_k}\right]+2\alpha\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)\right\}-\right. \\
&-\ln\left(\frac{8n_s\alpha^2 l}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)\right)- \\
&-\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)+2\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)- \\
&-3\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)+4/3\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)- \\
&\left.-3\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos\left[4\theta_{\varepsilon_k}\right]-1\right)+2/3\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\right)\right]
\end{aligned}
\tag{4.29}
$$

Calculating denominator of RA correlation function:

$$
A_{\varepsilon_k,\varepsilon_k,i,l}-\left(1-2\delta_{l0}\right)\left(1-2\delta_{i2}\right)B_{\varepsilon_k,\varepsilon_k,i,l} = -\frac{g}{4}q^2-\frac{g}{4}S''_{i,l}
\tag{4.30}
$$

$$
\begin{aligned}
S''_{i,l} = \frac{2}{D}\left[\left|\varepsilon_k\right|\cos\left(\theta_{\varepsilon_k}\right)+\Delta\sin\left(\theta_{\varepsilon_k}\right)\right]-\frac{8n_s\alpha\left(\left(1+\alpha^2\right)\cos\left[2\theta_{\varepsilon_k}\right]+2\alpha\right)}{g\left(1+2\alpha\cos\left[2\theta_{\varepsilon_k}\right]+\alpha^2\right)^2}\times \\
\times\left[\frac{1}{3}\left(1-4\delta_{l0}\right)\left(1-2\delta_{i3}-2\delta_{i0}\right)-1\right]
\end{aligned}
\tag{4.31}
$$

RR correlation function can be calculating by integration one fraction:

$$
\begin{aligned}
&\int \frac{d^2q}{(2\pi)^2}\sin^2(\theta_{\varepsilon_k})\sum_{i,l}\frac{1}{A_{\varepsilon_k,\varepsilon_k,i,l}-(1-2\delta_{l0})(1-2\delta_{i2})B_{\varepsilon_k,\varepsilon_k,i,l}}=\\
&=-\frac{1}{\pi g}\sum_{i,l}\sin^2(\theta_{\varepsilon_k})\ln\left[q^2+S''_{i,l}\right]\Bigg|_{1/L}^{1/l}=-\frac{2}{\pi g}\sin^2(\theta_{\varepsilon_k})\left[2\ln\left(\frac{L}{l}\right)-\right.\\
&-\ln\left(\frac{16l^2n_s\alpha\left((1+\alpha^2)\cos[2\theta_{\varepsilon_k}]+2\alpha\right)}{g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}+\frac{2l^2}{D}\left[|\varepsilon_k|\cos(\theta_{\varepsilon_k})+\Delta\sin(\theta_{\varepsilon_k})\right]\right)-\\
&-3\ln\left(\frac{16l^2n_s\alpha\left((1+\alpha^2)\cos[2\theta_{\varepsilon_k}]+2\alpha\right)}{3g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}+\frac{2l^2}{D}\left[|\varepsilon_k|\cos(\theta_{\varepsilon_k})+\Delta\sin(\theta_{\varepsilon_k})\right]\right)-\\
&\left.-3\ln\left(\frac{32l^2n_s\alpha\left((1+\alpha^2)\cos[2\theta_{\varepsilon_k}]+2\alpha\right)}{3g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}+\frac{2l^2}{D}\left[|\varepsilon_k|\cos(\theta_{\varepsilon_k})+\Delta\sin(\theta_{\varepsilon_k})\right]\right)\right]
\end{aligned}
\tag{4.32}
$$

So we get fluctuations:

$$
\begin{aligned}
K_2(E,E,r)\approx\ &\frac{\rho_0^2}{8\pi g}\left[2\ln\left(\frac{L}{l}\right)\cos(2\theta_{\varepsilon_k})-7\cos^2(\theta_{\varepsilon_k})\ln\left(\frac{l^2n_s\alpha\left((1+\alpha^2)\cos[2\theta_{\varepsilon_k}]+2\alpha\right)}{g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}\right)-\right.\\
&-\sin^2(\theta_{\varepsilon_k})\ln\left(\frac{16l^2n_s\alpha\left((1+\alpha^2)\cos[2\theta_{\varepsilon_k}]+2\alpha\right)}{g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}+\frac{2l^2}{D}\left[|\varepsilon_k|\cos(\theta_{\varepsilon_k})+\Delta\sin(\theta_{\varepsilon_k})\right]\right)-\\
&-3\sin^2(\theta_{\varepsilon_k})\ln\left(\frac{16l^2n_s\alpha\left((1+\alpha^2)\cos[2\theta_{\varepsilon_k}]+2\alpha\right)}{3g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}+\frac{2l^2}{D}\left[|\varepsilon_k|\cos(\theta_{\varepsilon_k})+\Delta\sin(\theta_{\varepsilon_k})\right]\right)-\\
&-3\sin^2(\theta_{\varepsilon_k})\ln\left(\frac{32l^2n_s\alpha\left((1+\alpha^2)\cos[2\theta_{\varepsilon_k}]+2\alpha\right)}{3g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}+\frac{2l^2}{D}\left[|\varepsilon_k|\cos(\theta_{\varepsilon_k})+\Delta\sin(\theta_{\varepsilon_k})\right]\right)-\\
&-\ln\left(\frac{8n_s\alpha^2l^2}{g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}\left(\cos[4\theta_{\varepsilon_k}]-1\right)\right)-\\
&-\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos[4\theta_{\varepsilon_k}]-1\right)+2\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}\right)-\\
&-3\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos[4\theta_{\varepsilon_k}]-1\right)+4/3\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}\right)-\\
&\left.-3\ln\left(\frac{8n_s\alpha l^2\left(\alpha\left(\cos[4\theta_{\varepsilon_k}]-1\right)+2/3\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)\right)}{g\left(1+2\alpha\cos[2\theta_{\varepsilon_k}]+\alpha^2\right)^2}\right)\right]
\end{aligned}
\tag{4.33}
$$

For L so large that $\ln\left(\frac{L}{l}\right)$ is rather bigger then $\ln\left(L^2S_0\right)$ we can write:

$$
\left\langle\left[\rho(E,r)-\langle\rho(E,r)\rangle\right]^2\right\rangle=\frac{\rho_0^2}{4\pi g}\ln\left(\frac{L}{l}\right)\sinh^2(\psi) \tag{4.34}
$$

If $L^2 S_0 \gg 1$ then we get answer without impurities:

$$\left\langle [\rho(E,r) - \langle\rho(E,r)\rangle]^2 \right\rangle = \frac{2\rho_0^2}{\pi g} \ln\left(\frac{L}{l}\right) \left(\sinh^2(\psi) + 1\right) \tag{4.35}$$

If we consider E close to E_g:

$$\frac{\left\langle [\rho(E,r) - \langle\rho(E,r)\rangle]^2 \right\rangle}{\langle\rho(E,r)\rangle^2} = \frac{1}{\pi g} \frac{|F''(\psi_g)| \cosh\psi_g \sinh^2(\psi_g)}{\frac{E-E_g}{\Delta}} \ln\left(\frac{L}{l}\right) \tag{4.36}$$

That's mean that there is self-averaging of DOS when $\frac{E-E_g}{\Delta} \gg \frac{|F''(\psi_g)| \cosh\psi_g \sinh^2(\psi_g)}{\pi g} \ln\left(\frac{L}{l}\right)$. And there isn't self-averaging when $\frac{E-E_g}{\Delta} \ll \frac{|F''(\psi_g)| \cosh\psi_g \sinh^2(\psi_g)}{\pi g} \ln\left(\frac{L}{l}\right)$. It is worth to say that another restriction for closeness to spectrum edge $\frac{(E-E_g)}{\Delta} \gg \frac{\eta\mu}{2\pi g} \frac{\sinh^3[\psi_g] \cosh^2\psi_g}{(1-\mu\cosh[2\psi_g])^2}$ is weaker condition, so this condition does not interfere with the existence of non-self-averaging case.

Chapter 5

CONCLUSION

So we calculated correlators $\left\langle \left(w^{\alpha\beta}_{\varepsilon_n,-\varepsilon_m}(q)\right)_{il} \left(\bar{w}^{\gamma\sigma}_{-\varepsilon_k,\varepsilon_p}(-q)\right)_{il} \right\rangle_{S^{(2)}}$ and we have used it in calculation correction of DOS and finding fluctuation. We find sharp edge of spectrum in correction and this results can be used not so close to the edge of spectre and we must find other orders to calculate DOS near edge. We also found out that fluctuations is equal to zero when DOS is equal to zero. We also researched what fluctuations are equal to when DOS is not equal to zero, and show that there is violating of self-averaging of DOS close to the edge.

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