Yan V Fyodorov
Chair in Disordered Systems
Department of Mathematics
King's College London
Strand, London WC2R 2LS, UK
E-mail: yan.fyodorov@kcl.ac.uk

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Referee report on MASTER's THESIS

Influence of the processes of resonant scattering on critical Josephson current through Anderson insulator

by Nikolay Kishmar

The main goal of the thesis is to provide an analysis of the mean value of Josephson current through a disordered junction where electron states are fully localized due to disorder.

The thesis starts with outlining the functional integral approach to statistics of the wavefunction tails in 1D and quasi1D disordered systems developed in the work [13] by Ivanov et al. In particular, it goes beyond [13] in extracting the value of the derivative of a factor in the distribution at the origin (inaccurately denoted as "incline", whereas "slope" or "gradient" would be more appropriate notation). Such information is efficiently used in the consideration of the mean critical current accross disordered Josephson junction, which is considered next in the framework of Beenakker's approach [5].

To simplify the consideration the thesis addresses only a single-channel scattering accross the disordered junction, characterized in terms of a 2×2 scattering matrix, of which the current under consideration only depends on the frequency-dependent off-diagonal term (transmission $|S_{12}(\omega)|$). The mean current is studied analytically assuming that the typical broadening of the energy levels Γ_{typ} in the junction region is much smaller than the superconducting gap Δ_0 (which holds in long enough junctions). In the opposite limit $\Delta_0 \ll \Gamma_{typ}$ the evaluation is performed approximately, but the result is checked to give parametrically correct order of magnitude. The relevant length scale separating two regimes is identified with the so-called Mott scale L_M.

The central role in the subsequent evaluation is played by eq.(2.2) representing the transmission coefficient in terms of a summation over(localized) eigenfunctions and associated energy levels in the disordered region. This expression is subdivided into diagonal

and off-diagonal contribution. Approximating the level widths by first-order perturbation theory formula (2.7) (its origin is not properly discussed), and using the wavefunction statistics as presented in the beginning of the thesis, one easily evaluates the diagonal contribution to the mean transmission. Evaluating off-diagonal contribution requires more care as one has to account for the Mott-type level hybridization and proper statistics of the associated coupling element. Such analysis is accurately and convincingly presented in the Section (2.2).

The subsequent analysis is done separately for regimes below and above the Mott scale. In the former regime the overall result is expected to be correct up to a factor of order of unity. Most interesting and involved is the analysis in the case of a long sample. Interestingly, one has to further distinguish between two regimes: $L_M \ll L \ll L_M^2/\xi$ and $L \gg L_M^2/\xi$, with ξ being the localization length. Analysis requires employing various tricks for accurately estimating both the diagonal and off-diagonal contributions. Even after serious simplifications the final result requires numerical evaluation of ensuing integrals. The final formulas are reasonably simple, and show that the mean critical current is dominated by rare realizations of disorder, reflected by the corresponding factor in the exponential.

Summarizing, the present thesis reports results of original research on the topic of considerable interest in theoretical physics. It elucidates the role of rare events dominating the mean value of the Josephson current through the disordered junction, and boosts our present understanding of effects in disordered junctions where Anderson localization effects play important role. Computations are technically involved and require high analytical skills. The work is done at high standards. The presentation is however sketchy in places and could be more consistent (e.g. discussing the assumptions behind the formulas eq.(2.2) and (2.7) taken for granted), but this point is only a minor criticism. I evaluate the quality of the thesis as 8 points of 10-point scale, and do not hesitate to confirm that the author of the thesis fully deserves to be awarded the Master's degree.

Yours sincerely,

Yan V Fyodorov