Unusual Hall effects and linear magnetoconductivity in fermion systems with spin-momentum locking

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Introduction

- From our college years we are accustomed to classical definition of the Hall effect, namely the electric field **E**, passed electric current **j** and the magnetic field **B** are mutually othogonal to each other,

$$\mathbf{j}_{\mathrm{H}} \propto [\mathbf{E} \times \mathbf{B}] \, .$$

  The effect is due to the magnetic part of the Lorentz force which curves fermion's trajectories.

- We will discuss other possibilities of the Hall effect, in which case only the electric current and electric field are othogonal, while the magnetic field is in plane with the two. To the best of my knowledge such effect has been experimentally observed for the first time in 2018.

- We will also discuss linear magnetoconductivity in magnetic metals. It appeared that besides the Onsager relation there has not been any knowledge of the mechanism behind the effect. It also appeared that despite century of working with ferromagnets, there has not been any reports of the experimental observation of the effect until very recently. Was it the elusive Joe effect, who knows.

Spin

- Spin has a geometric phase. If we consider evolution in parameter space defined by $i = 1..N$ (for example, time), the phase is

$$\chi = -\mathrm{Im}\ln\left[\langle i=1|i=2\rangle\ldots\langle i=n|i=n+1\rangle\ldots\langle i=N-1|i=N\rangle\right],$$

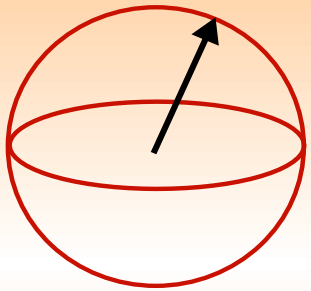
  where $|i\rangle$ is the ket corresponding to the spin's state at step i.
- Adiabatic approximation: the overlap of neighboring states is smooth.
- For example, spin $\mathbf{S}$ on the unit sphere.

$$H = -g\mathbf{B}\cdot\mathbf{S},$$

where $\mathbf{B}$ is the magnetic field.

$$|\theta,\phi\rangle = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right)e^{i\phi} \end{bmatrix},$$

where θ, ϕ are polar and azimuthal angles.



- Evolve it along a closed trajectory:
  (1) from $\mathbf{S} = S\mathbf{e}_z$ to $\mathbf{S} = S\mathbf{e}_y$;
  (2) from $\mathbf{S} = S\mathbf{e}_y$ to $\mathbf{S} = -S\mathbf{e}_z$;
  (3) from $\mathbf{S} = -S\mathbf{e}_z$ to $\mathbf{S} = -S\mathbf{e}_y$;
  (4) from $\mathbf{S} = -S\mathbf{e}_y$ to $\mathbf{S} = S\mathbf{e}_z$;
- The phase along this path is

$$\langle +z|+y\rangle = \frac{1}{\sqrt{2}},\quad \langle +y|-z\rangle = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{2}},\quad \langle -z|-y\rangle = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{2}},\quad \langle -y|+z\rangle = \frac{1}{\sqrt{2}}.$$

$$\chi = -\mathrm{Im}\ln\left[e^{i\frac{\pi}{2}}e^{i\frac{\pi}{2}}\right] = -\pi.$$

- The spin didn't come back to its initial state.

Phase

- This phase can't be gauged away. Hence, if speaking of electron systems, the phase can potentially result in electric currents (recall that imaginary part of the wave function corresponds to the current).
- In solids the parameter space is the momentum space, for example, $\lambda \in (k_x, k_y)$ in 2D.

$$\chi = -\mathrm{Im} \sum_{\lambda} \ln \langle u_\lambda | u_{\lambda + d\lambda} \rangle \to -\mathrm{Im} \oint_{\mathrm{P}} \langle u_\lambda | \partial_\lambda u_\lambda \rangle d\lambda = \oint_{\mathrm{P}} \langle u_\lambda | i \partial_\lambda u_\lambda \rangle d\lambda$$
$$= \int_{\mathrm{S}} \Omega(\lambda) dS.$$

Stockes theorem was used and $\Omega = -2\mathrm{Im}\langle \partial_x u | \partial_y u \rangle$ is the Berry curvature.

- In order to operate with the spin's phase we need coupling of the momentum with the spin through spin-momentum locking, i.e. spin-orbit coupling or momentum dependent exchange in magnets.

Anomalous velocity

- The Berry curvature $\mathbf{\Omega_k}$ affects the semi-classical equations of motions of electron wave-packet.

$$\dot{\mathbf{r}} = \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \mathbf{\Omega_k},$$
$$\dot{\mathbf{k}} = e\mathbf{E} + \frac{e}{c}\dot{\mathbf{r}} \times \mathbf{B},$$

  where $\epsilon_{\mathbf{k}}$ is electron's dispersion, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields correspondingly.

- The Berry curvature $\mathbf{\Omega_k}$ plays a role of an effective magnetic field in momentum space.
- The Berry curvature $\mathbf{\Omega_k}$ introduces what is called the anomalous velocity, i.e. $\dot{\mathbf{r}}_{\mathrm{anomalous}} \propto e\mathbf{E} \times \mathbf{\Omega_k}$.
- For a review see D. Xiao, M.C. Chang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010), *Berry phase effects on electronic properties*.
- $$\mathbf{j}_{\mathrm{AHE}} = e^2 \sum_s \int_{\mathbf{k}} \left[\mathbf{\Omega}_{s;\mathbf{k}} \times \mathbf{E}\right] \mathcal{F}_{s;\mathbf{k}}, \tag{1}$$

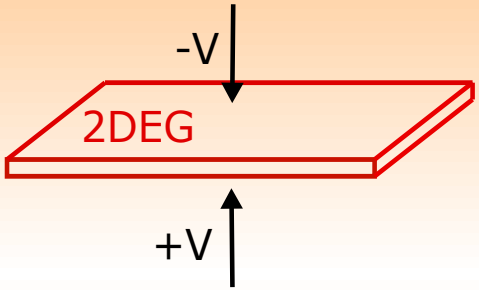
  $s-$ band index, $\mathcal{F}_{s;\mathbf{k}}$ is the Fermi-Dirac distribution function.

Spin-orbit coupling

- Due to uncompensted electric potential gradients.
- Rashba spin-orbit coupling

$$H_{\mathrm{R}} = \lambda \left(k_x \sigma_y - k_y \sigma_x \right) ,$$

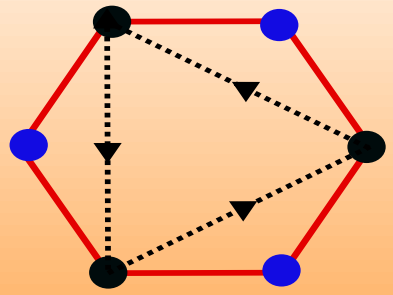
where λ is a constant.



- C_{3v} spin-orbit coupling

$$H_{3v} = \alpha k_x \left(k_x^2 - 3k_y^2 \right) \sigma_z ,$$

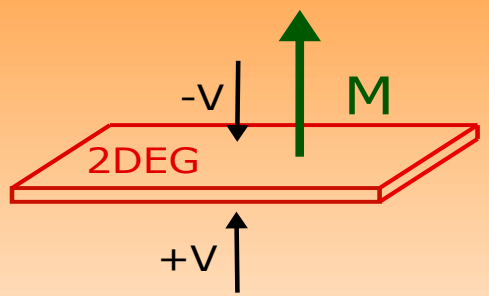
where α is a constant.



Anomalous Hall effect in ferromagnets

- Conventional model of 2DEG with Rashba spin-orbit coupling and ferromagnetic exchange interaction.

$$H_{\mathrm{FM}} = \frac{\mathbf{k}^2}{2m} + \lambda \left(k_x \sigma_y - k_y \sigma_x\right) + \Delta_{\mathbf{k}} \sigma_z .$$



- The spinor structure corresponding to $\epsilon_{\mathbf{k};\pm} = \frac{\mathbf{k}^2}{2m} \pm \sqrt{(\lambda k)^2 + \Delta_{\mathbf{k}}^2}$

$$\Psi_{\mathbf{k},+} = \begin{bmatrix} \cos\left(\frac{\xi_{\mathbf{k}}}{2}\right) e^{i\phi_{\mathbf{k}}}, \\ -\sin\left(\frac{\xi_{\mathbf{k}}}{2}\right) \end{bmatrix}, \quad \Psi_{\mathbf{k},-} = \begin{bmatrix} \sin\left(\frac{\xi_{\mathbf{k}}}{2}\right) e^{i\phi_{\mathbf{k}}} \\ \cos\left(\frac{\xi_{\mathbf{k}}}{2}\right) \end{bmatrix},$$

where $\cos(\xi_{\mathbf{k}}) = \frac{\Delta_{\mathbf{k}}}{\sqrt{\Delta_{\mathbf{k}}^2 + \lambda^2 k^2}}$ and $\phi_{\mathbf{k}} = \arctan\left(\frac{k_y}{k_x}\right)$ is the phase.

- For a ferromagnet $\Delta_{\mathbf{k}} = M_z$, and there is an anomalous Hall effect in this model given by

$$\mathbf{j}_{\mathrm{AHE}} \propto [\mathbf{E} \times \mathbf{M}] = M_z \left[\mathbf{E} \times \mathbf{e}_z\right].$$

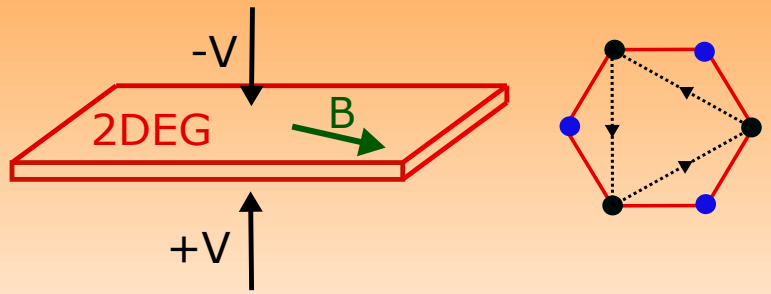
- For a review see N. Nagaosa, J. Sinova, S. Onoda, A. H. MacDonald, and N. P. Ong, Rev. Mod. Phys. **82**, 1539 (2010), *Anomalous Hall effect*.

Magnetic field driven C_{3v} in-plane Hall effect

- Consider a model of 2DEG with Rashba spin-orbit coupling and C_{3v} spin-orbit coupling and in-plane magnetic field,

$$H = \frac{\mathbf{k}^2}{2m} + \lambda\left(k_x\sigma_y - k_y\sigma_x\right) + \alpha k_x\left(k_x^2 - 3k_y^2\right)\sigma_z + h_x\sigma_x + h_y\sigma_y,$$

where $h_i = \frac{1}{2}g\mu_{\mathrm{B}}B_i$.



- There is an anomalous Hall effect in this model

$$\mathbf{j}_{\mathrm{AHE}} \propto B_y(B_y^2 - 3B_x^2)\left[\mathbf{E}\times\mathbf{e}_z\right] = B_{\parallel}^3\cos(3\phi)\left[\mathbf{E}\times\mathbf{e}_z\right].$$

Anisotropy relative to the crystal structure when the magnetic field is tuned in the plane.

Signatures of the Hall effect capturing the C_{3v} symmetry has been recently observed in ferromagnetic system CuPt/CoPt bilayer. L. Liu et al, *Crystal Symmetry-Dependent In-Plane Hall Effect* Nano Letters 2024.

- If $\alpha k_x\left(k_x^2 - 3k_y^2\right)\sigma_z \to \lambda_{\mathrm{D}}k_x\sigma_z$ then

$$\mathbf{j}_{\mathrm{AHE}} \propto B_y\left[\mathbf{E}\times\mathbf{e}_z\right].$$

This effect has been experimentally observed in 2018 in $ZrTe_5$ material.

In-plane Hall effect. Experiment.

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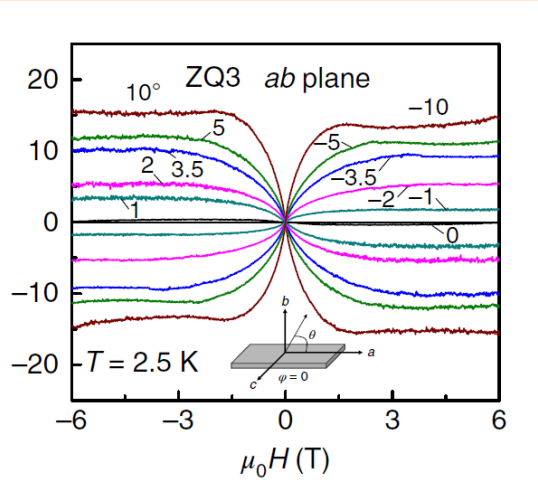
https://doi.org/10.1038/s41567-018-0078-z

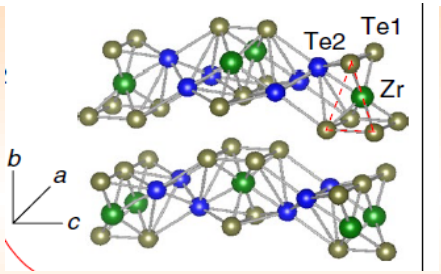
Anomalous Hall effect in $ZrTe_5$

NATURE PHYSICS | VOL 14 | MAY 2018 | 451–455 |

Tian Liang[1,2,3,7*], Jingjing Lin[1,7], Quinn Gibson[4], Satya Kushwaha[4], Minhao Liu[1], Wudi Wang[1], Hongyu Xiong[2,3], Jonathan A. Sobota[2,3,5], Makoto Hashimoto[6], Patrick S. Kirchmann[3], Zhi-Xun Shen[2,3], R. J. Cava[4] and N. P. Ong[1*]

Next, we focus on the contribution from the in-plane component of **H** (in the *ac* plane). With **H** in plane and at an angle φ to **a**, we observe remarkably large AHE signals at each value of φ except when $\mathbf{H}||a$ ($\varphi = 0$) (see Fig. 3d–f). The AHE contribution





$ZrTe_5$ has an orthorhombic layered structure with space group *Cmcm* (D_{2h}^{17})[25] (inset in Fig. 1a). The $ZrTe_6$ triangular prisms (depicted as the red dashed lines) form one-dimensional chains of $ZrTe_3$ running along the *a* axis. The chains are connected by additional Te ions, which also form zigzag chains along the *a* axis and extend along the *c* axis. As a result, they define quasi-two-dimensional layers that stack along the *b* axis via van der Waals interactions to form the 3D crystal. The magnitude of the van der Waals interaction along the *b* axis is very small, comparable to that in graphite[25]. Therefore, both the two-dimensional single layer and the 3D bulk crystals of $ZrTe_5$ are of interest. A monolayer of $ZrTe_5$ is predicted to feature the quan-

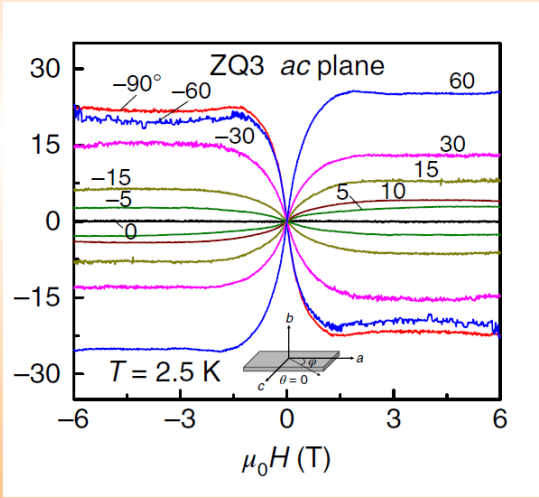
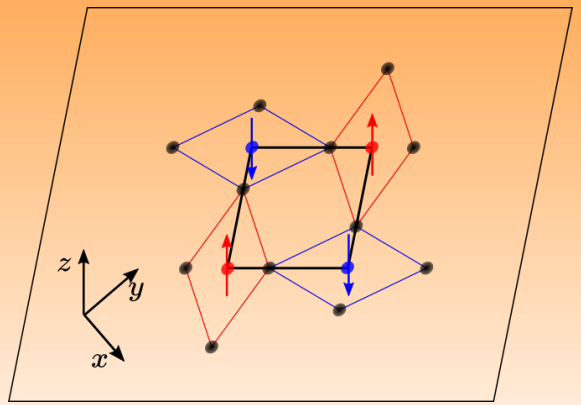
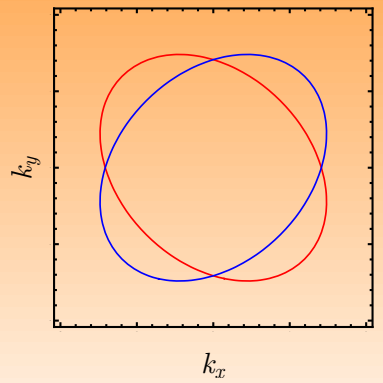


Figure: Anomalous Hall effect in ZrTe5. Left: regular. Right: in-plane when magnetic field is in plane with the electric current and transverse to the current voltage drop. $ZrTe_5$ is quasi two-dimensional.

d-wave Hall effect in antiferromagnets

- Square lattice antiferromagnet with Neel order in $z-$ direction. RuO_2 for example.





For a review see Phys. Rev. X **12**, 031042 (2022), Phys. Rev. X **12**, 040501 (2022).
Hamiltonian of the model is

$$H = \frac{\mathbf{k}^2}{2m} + \lambda(k_x\sigma_y - k_y\sigma_x) + \beta\sigma_z k_x k_y + h_x\sigma_x + h_y\sigma_y,$$

where $h_i = \frac{1}{2} g\mu_{\mathrm{B}} B_i$.

- There is anomalous Hall effect

$$\mathbf{j}_{\mathrm{AHE}} \propto \sigma_{\mathrm{DWHE}}\beta B_x B_y \left[\mathbf{e}_z \times \mathbf{E}\right],$$

which has the $d-wave$ symmetry in magnetic field. In addition,

$$\mathbf{j}_{\mathrm{LMC}} \propto \beta B_z \left(E_x\mathbf{e}_y + E_y\mathbf{e}_x\right),$$

which is the linear magnetoconductivity. It is allowed by the Onsager relation.

- D.L. Vorobev and V.A. Zyuzin, arxiv 2311.04890.

LMC in ferromagnetic Weyl metals

- Onsager relation for the conductivity tensor is $\sigma_{ij}(\mathbf{B}, \mathbf{M}) = \sigma_{ji}(-\mathbf{B}, -\mathbf{M})$. Therefore, if the time-reversal symmetry is preserved in the system, longitudinal and non-Hall parts must be $\propto B^2$.
- If there is a magnetization $\mathbf{M}$ in the system, i.e. the system is a ferromagnet, the most straightforward structure of the linear magnetoconductivity is

$$\delta \mathbf{j}_{\mathrm{LMC}} = \alpha_1(\mathbf{E} \cdot \mathbf{M})\mathbf{B} + \alpha_1(\mathbf{E} \cdot \mathbf{B})\mathbf{M} + \alpha_2(\mathbf{B} \cdot \mathbf{M})\mathbf{E}.$$

- Simplest model of magnetic Weyl metal is

$$H_{\pm} = \pm v(\mathbf{p} \cdot \sigma) \pm \beta_1 M_z p_z + \beta_2 M_z p_z^2 \sigma_z$$

has the effect. All three terms in the current $\delta \mathbf{j}_{\mathrm{LMC}}$ are present due to the β_1 and β_2. Somehow only the last two have been recently experimentally observed in magnetic Weyl semimetal $Co_3Sn_2S_2$ in PRL 126, 236601 (2021).

Main message

- Why worry about Hall effects? They don't disappear if the current is passed along the main axes of the crystal. It is the magnetic field which can nullify Hall effects. Recall that in-plane Hall effect is $\propto B_y\,[\mathbf{E}\times\mathbf{e}_z]$, C_{3v} in-plane Hall effect is $\propto B_y(B_y^2-3B_x^2)\,[\mathbf{E}\times\mathbf{e}_z]$, and d-wave Hall effect is $\propto B_xB_y\,[\mathbf{E}\times\mathbf{e}_z]$.
- Other transverse responses such as linear magnetoconductivity might disappear if the current is passed along the main axes. This is true for $\mathbf{j}_{\mathrm{LMC}}\propto B_z\left(E_x\mathbf{e}_y+E_y\mathbf{e}_x\right)$ when the current is passed at $\frac{\pi}{4}$.
- Can we utilize these effects to map out the symmetry of the crystal structure of the magnetic metal?