

Лекция №1 по теории сверхпроводимости.

Часть 1:

Краткое введение о различных
сверхпроводниках

Различные семейства сверхпроводящих материалов

Простые металлы

Elemental metals

Сплавы металлов

Metal alloys

Медь-оксидные ВТСП

Copper-oxides HTSC

MgB₂

Magnesium di-boride

Железо-мышьяк

Ferro-pnictides

Рутенаты

Rhutenium-based materials

Сверхпроводники с аномально малой плотностью электронов

Extra-low-density superconductors

Двумерные сверхпроводники

2D superconductors

Сильно неупорядоченные сверхпроводники

Strognly disordered superconductors

PERIODIC TABLE OF SUPERCONDUCTING ELEMENTS

from Yamashita T, Nakajima K, Chen J, Buzea C,
Superconductors - Scientific Basics and Engineering Applications
 (Springer-Verlag, Heidelberg) 2002 to appear

| IA | | | | | | | | | | IIB | | | | | | | | | | IIB | | | | | | | | | | VI | | | | | | | | | | VI | | | | | | | | | | VI | | | | | | | | | | VIII | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 1 H Hydrogen | | | | | | | | | | 2 He Helium | | | | | | | | | | 3 Li Lithium 7 K 28 GPa | | | | | | | | | | 4 Be Beryllium 0.008 K 9 K 8 m | | | | | | | | | | 5 B Boron 11.2 K 250 GPa | | | | | | | | | | 6 C Carbon 52 K charge doped | | | | | | | | | | 7 N Nitrogen | | | | | | | | | | 8 O Oxygen 0.5 K 100 GPa | | | | | | | | | | 9 F Fluorine | | | | | | | | | | 10 Ne Neon | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 Na Sodium | | | | | | | | | | 12 Mg Magnesium | | | | | | | | | | 13 Al Aluminum 1.17 K 3.9 K 8 m | | | | | | | | | | 14 Si Silicon 0.5 K 12 GPa | | | | | | | | | | 15 P Phosphorus 5.9 K 17 GPa | | | | | | | | | | 16 S Sulfur 17 K 100 GPa | | | | | | | | | | 17 Cl Chlorine | | | | | | | | | | 18 Ar Argon | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 19 K Potassium | | | | | | | | | | 20 Ca Calcium 15 K 150 GPa | | | | | | | | | | 21 Sc Scandium | | | | | | | | | | 22 Ti Titanium 0.4 K | | | | | | | | | | 23 V Vanadium 5.4 K 17.2 K 120 GPa | | | | | | | | | | 24 Cr Chromium film | | | | | | | | | | 25 Mn Manganese film | | | | | | | | | | 26 Fe Iron 2 K 21 GPa | | | | | | | | | | 27 Co Cobalt | | | | | | | | | | 28 Ni Nickel | | | | | | | | | | 29 Cu Copper | | | | | | | | | | 30 Zn Zinc 0.85 K | | | | | | | | | | 31 Ga Gallium 7.8 K 8.4 K 8 m | | | | | | | | | | 32 Ge Germanium 0.4 K 11.5 GPa | | | | | | | | | | 33 As Arsenic 2.7 K 24 GPa | | | | | | | | | | 34 Se Selenium 7 K 13 GPa | | | | | | | | | | 35 Br Bromine 1.4 K 150 GPa | | | | | | | | | | 36 Kr Krypton | | | | | | | | | |
| 37 Rb Rubidium | | | | | | | | | | 38 Sr Strontium 4 K 50 GPa | | | | | | | | | | 39 Y Yttrium | | | | | | | | | | 40 Zr Zirconium 0.6 K | | | | | | | | | | 41 Nb Niobium 0.25 K 0.7 K 4.5 GPa | | | | | | | | | | 42 Mo Molybdenum 0.92 K | | | | | | | | | | 43 Tc Technetium 0.2 K | | | | | | | | | | 44 Ru Ruthenium 0.5 K | | | | | | | | | | 45 Rh Rhodium 0.005 mK | | | | | | | | | | 46 Pd Palladium film/rod | | | | | | | | | | 47 Ag Silver | | | | | | | | | | 48 Cd Cadmium 0.52 K | | | | | | | | | | 49 In Indium 3.4 K 4.2 K 8 m | | | | | | | | | | 50 Sn Tin 3.7 K 4.7 K 8 m | | | | | | | | | | 51 Sb Antimony 3.9 K 8.5 GPa | | | | | | | | | | 52 Te Tellurium 7.4 K 35 GPa | | | | | | | | | | 53 I Iodine 1.2 K 25 GPa | | | | | | | | | | 54 Xe Xenon | | | | | | | | | |
| 55 Cs Cesium 1.5 K 5 GPa | | | | | | | | | | 56 Ba Barium 5 K 15 GPa | | | | | | | | | | 57 La Lanthanum 0 K 12 K 14 GPa | | | | | | | | | | 58 Ce Cerium 1.7 K 8 m | | | | | | | | | | 59 Pr Praseodymium | | | | | | | | | | 60 Nd Neodymium | | | | | | | | | | 61 Pm Promethium | | | | | | | | | | 62 Sm Samarium | | | | | | | | | | 63 Eu Europium film | | | | | | | | | | 64 Gd Gadolinium | | | | | | | | | | 65 Tb Terbium | | | | | | | | | | 66 Dy Dysprosium | | | | | | | | | | 67 Ho Holmium | | | | | | | | | | 68 Er Erbium | | | | | | | | | | 69 Tm Thulium | | | | | | | | | | 70 Yb Ytterbium | | | | | | | | | | 71 Lu Lutetium 0.1 K | | | | | | | | | | | | | | | | | | | |
| 87 Fr Francium | | | | | | | | | | 88 Ra Radium | | | | | | | | | | 89 Ac Actinium | | | | | | | | | | 90 Th Thorium 1.4 K | | | | | | | | | | 91 Pa Protactinium 1.4 K | | | | | | | | | | 92 U Uranium 0.7 K 2.2 K 1 GPa | | | | | | | | | | 93 Np Neptunium | | | | | | | | | | 94 Pu Plutonium | | | | | | | | | | 95 Am Americium 1 K | | | | | | | | | | 96 Cm Curium | | | | | | | | | | 97 Bk Berkelium | | | | | | | | | | 98 Cf Californium | | | | | | | | | | 99 Es Einsteinium | | | | | | | | | | 100 Fm Fermium | | | | | | | | | | 101 Md Mendelevium | | | | | | | | | | 102 No Nobelium | | | | | | | | | | 103 Lr Lawrencium | | | | | | | | | | | | | | | | | | | |

Superconducting element
 Nonsuperconducting element
 Superconducting element only under pressure or in film form

Atomic number: 4
 Symbol: Be
 Name: Beryllium
 Critical temperature: 0.008 K
 Conditions: 9 K, 8 m

Maximum critical temperature under certain conditions
 Conditions

Review of superconducting properties of MgB2
 cond-mat/0108265

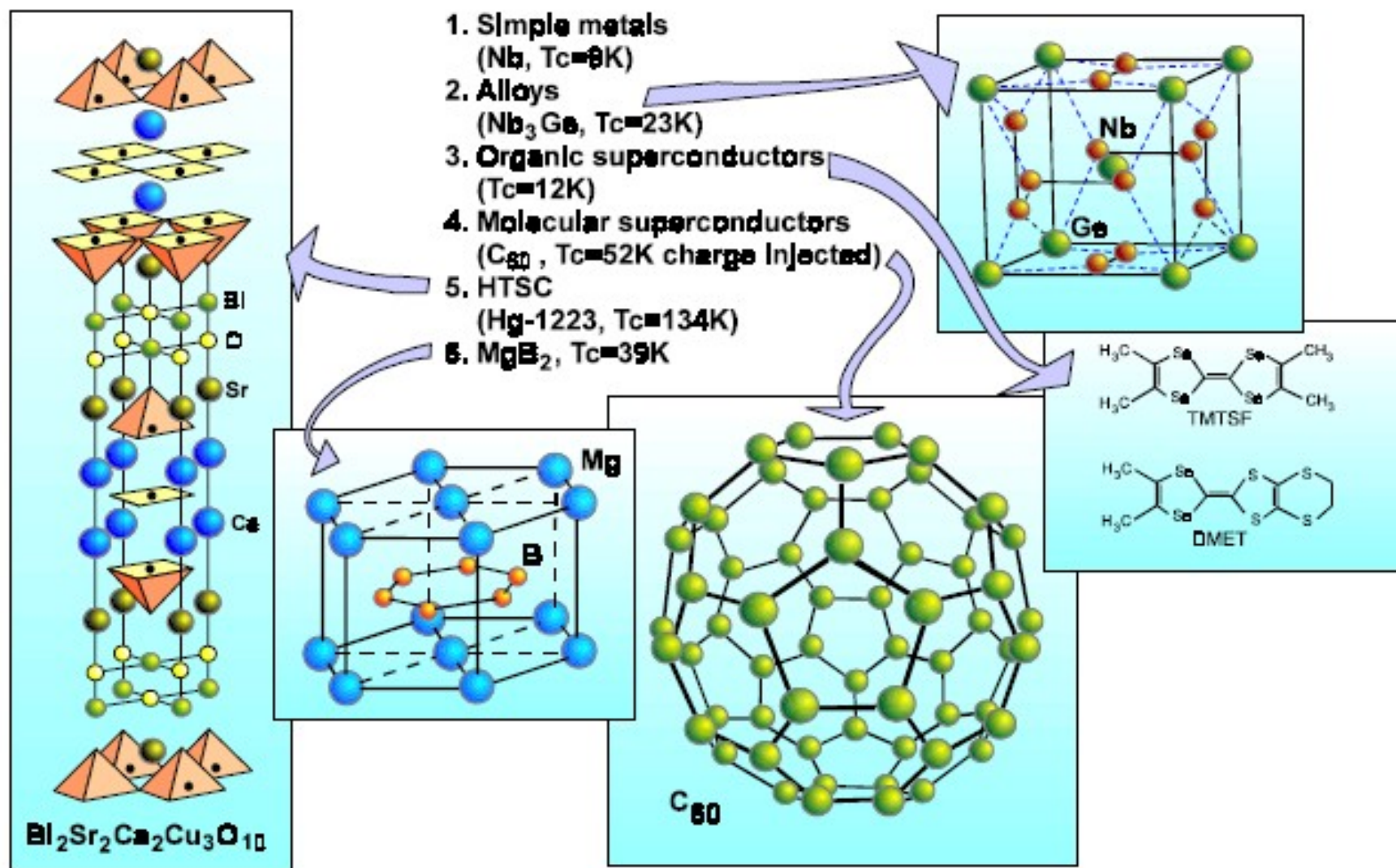


Figure 3. Comparison between the structures of different classes of superconductors

Простые сверхпроводящие металлы - элементы

1. Критические температуры и критические магнитные поля простых сверхпроводников [10].

| Элемент | T_c , К | $H_{cm}(0)$, Э |
|-----------------|--------------------|-----------------|
| Al | 1.175 ± 0.002 | 104.9 ± 0.3 |
| Be | 0.026 | |
| Cd | 0.517 ± 0.002 | 28 ± 1 |
| Ga | 1.083 ± 0.001 | 59.2 ± 0.3 |
| Hf | 0.128 | |
| Hg (α) | 4.154 ± 0.001 | 411 ± 2 |
| Hg (β) | 3.949 | 339 |
| In | 3.408 ± 0.001 | 281.5 ± 2 |
| Ir | 0.1125 ± 0.001 | 16 ± 0.05 |
| La (α) | 4.88 ± 0.02 | 800 ± 10 |
| La (β) | 6.00 ± 0.1 | 1096, 1600 |
| Lu | 0.1 | < 400 |
| Mo | 0.915 ± 0.005 | 96 ± 3 |
| Nb | 9.25 ± 0.02 | 2060 ± 50 |
| Os | 0.66 ± 0.03 | 70 |
| Pa | 1.4 | |
| Pb | 7.196 ± 0.006 | 803 ± 1 |
| Re | 1.697 ± 0.006 | 200 ± 5 |
| Ru | 0.49 ± 0.015 | 69 ± 2 |
| Sn | 3.722 ± 0.001 | 305 ± 2 |

| Элемент | T_c , К | $H_{cm}(0)$, Э |
|---------|---------------------|-----------------|
| Ta | 4.47 ± 0.04 | 829 ± 6 |
| Tc | 7.8 ± 0.01 | 1410 |
| Th | 1.38 ± 0.02 | 160 ± 3 |
| Ti | 0.40 ± 0.04 | 56 |
| Tl | 2.38 ± 0.04 | 178 ± 5 |
| V | 5.40 ± 0.05 | 1408 |
| W | 0.0154 ± 0.0005 | 1.15 ± 0.03 |
| Zn | 0.850 ± 0.01 | 54 ± 0.3 |
| Zr | 0.61 ± 0.15 | 47 |

Некоторые сверхпроводящие сплавы и соединения

Таблица 1.2. Критические температуры некоторых интерметаллических соединений и высокотемпературных сверхпроводников.

| Соединение | T_c , К |
|-----------------------------|-----------|
| Nb_3Sn | 18.1 |
| $(Nb_3Al)_4 + Nb_3Ge$ | 20 |
| $La_{0.925}Sr_{0.075}CuO_4$ | 34 |
| $YBa_2Cu_3O_7$ | 92.4 |
| $Bi_2Sr_2Ca_2Cu_3O_{10}$ | 111 |
| $Tl_2Sr_2Ca_2Cu_3O_{10}$ | 123 |
| $HgBa_2Ca_2Cu_3O_8$ | 133 |

Все это сверхпроводники
2 рода!
У них два разных критических поля

$$H_{c1} \ll H_{c2}$$



The Intriguing Superconductivity of Strontium Ruthenate
<http://physicstoday.scitation.org/doi/10.1063/1.1349611>

Экзотические материалы

The Intriguing Superconductivity of Strontium Ruthenate
<http://physicstoday.scitation.org/doi/10.1063/1.1349611>



Iron-based superconductors (FeSC)

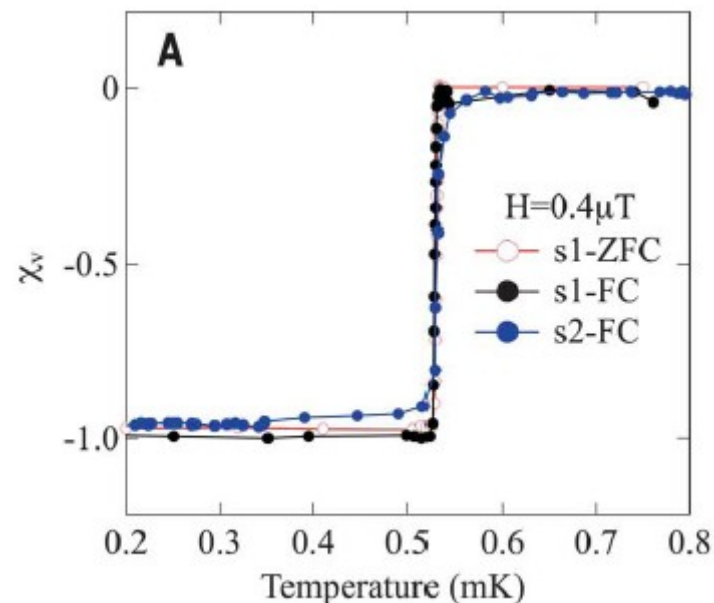
| <u>Oxypnictide</u> | T_c (K) |
|---|------------|
| $\text{LaO}_{0.89}\text{F}_{0.11}\text{FeAs}$ | 26[11] |
| $\text{LaO}_{0.9}\text{F}_{0.2}\text{FeAs}$ | 28.5[12] |
| $\text{CeFeAsO}_{0.84}\text{F}_{0.16}$ | 41[11] |
| $\text{SmFeAsO}_{0.9}\text{F}_{0.1}$ | 43[11][13] |
| $\text{La}_{0.5}\text{Y}_{0.5}\text{FeAsO}_{0.6}$ | 43.1[14] |
| $\text{NdFeAsO}_{0.89}\text{F}_{0.11}$ | 52[11] |
| $\text{PrFeAsO}_{0.89}\text{F}_{0.11}$ | 52[15] |
| ErFeAsO_{1-y} | 45[16] |

Сверхпроводимость почти без электронов

Evidence for bulk superconductivity in pure bismuth single crystals at ambient pressure

Prakash *et al.*, *Science* **355**, 52–55 (2017) 6 January 2017

$$n \approx 3 \times 10^{17} \text{ cm}^{-3}$$



Single gap superconductivity in doped SrTiO_3

<https://arxiv.org/abs/1703.04716>

Плотность носителей $10^{17} - 10^{20}$

$\epsilon > 20\,000$

Слабо допированный почти сегнетоэлектрик

2D superconductors

Supercond. Sci. Technol. 30 (2017) 013002 (37pp)

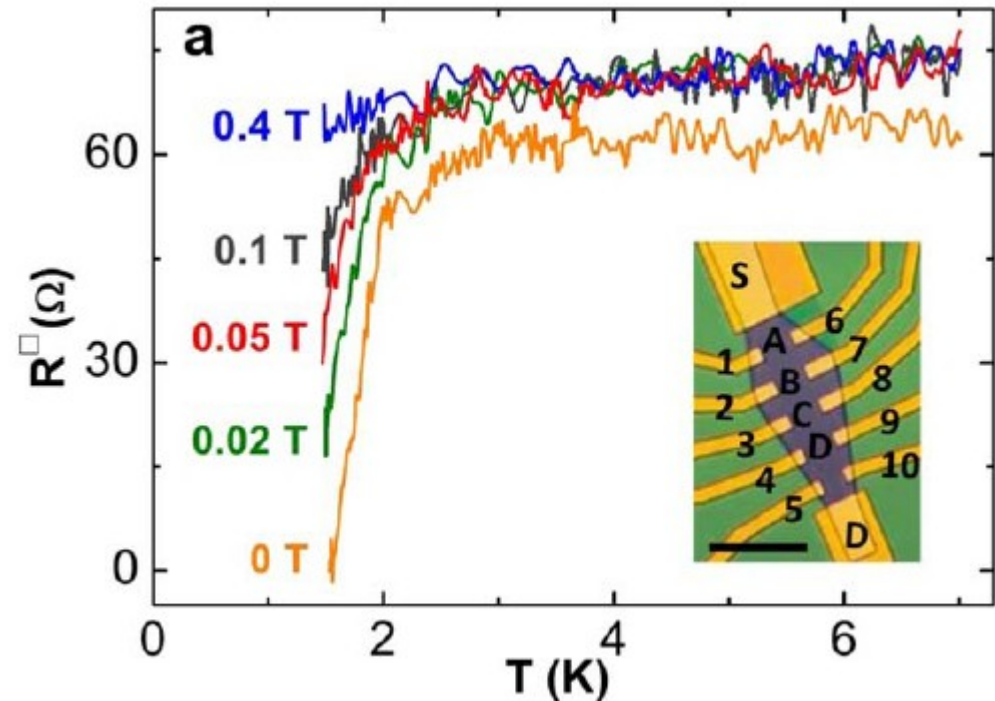
Takashi Uchihashi

Two-dimensional superconductors with atomic-scale thickness

$\text{LaAlO}_3/\text{SrTiO}_3$

Atomic-thickness interface between two insulators

MoS_2 – атомный слой



Очень сильно неупорядоченные сверхпроводники и квантовый переход сверхпроводник-изолятор

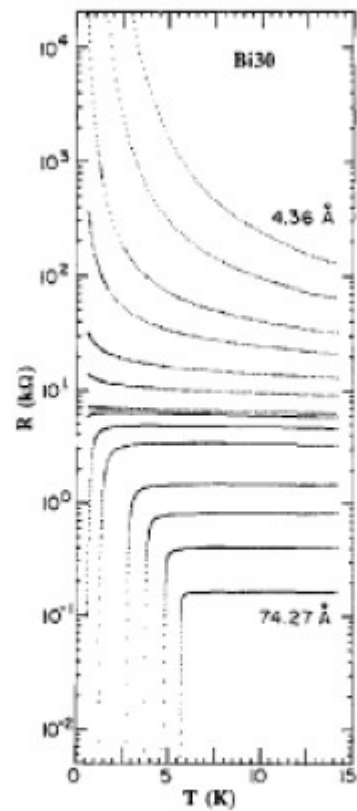
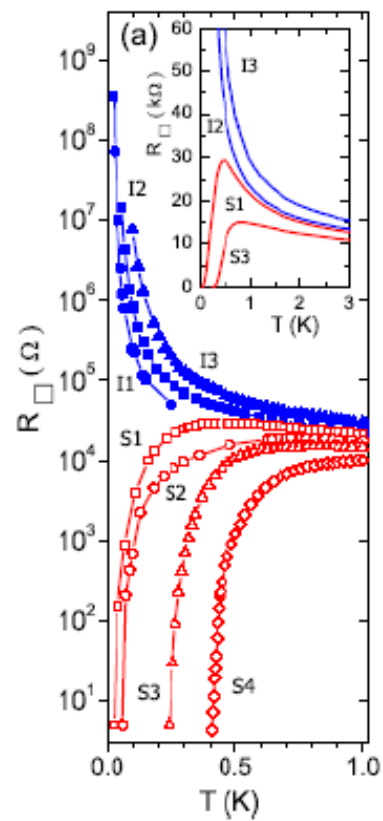


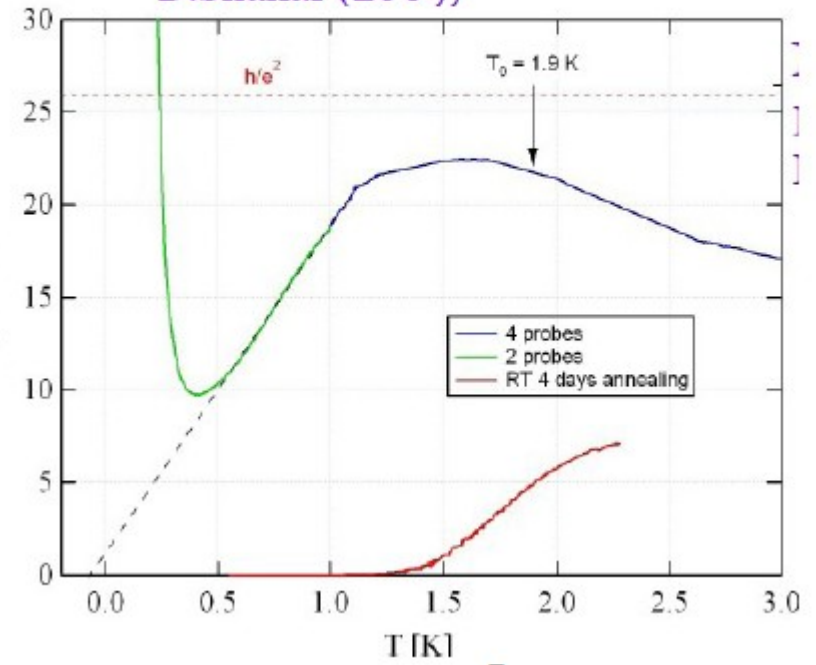
Fig. 5. Temperature dependence of the sheet resistance $R(T)$ for Bi films deposited onto Ge [15]. The films are considered to be homogeneous.



TiN thin films

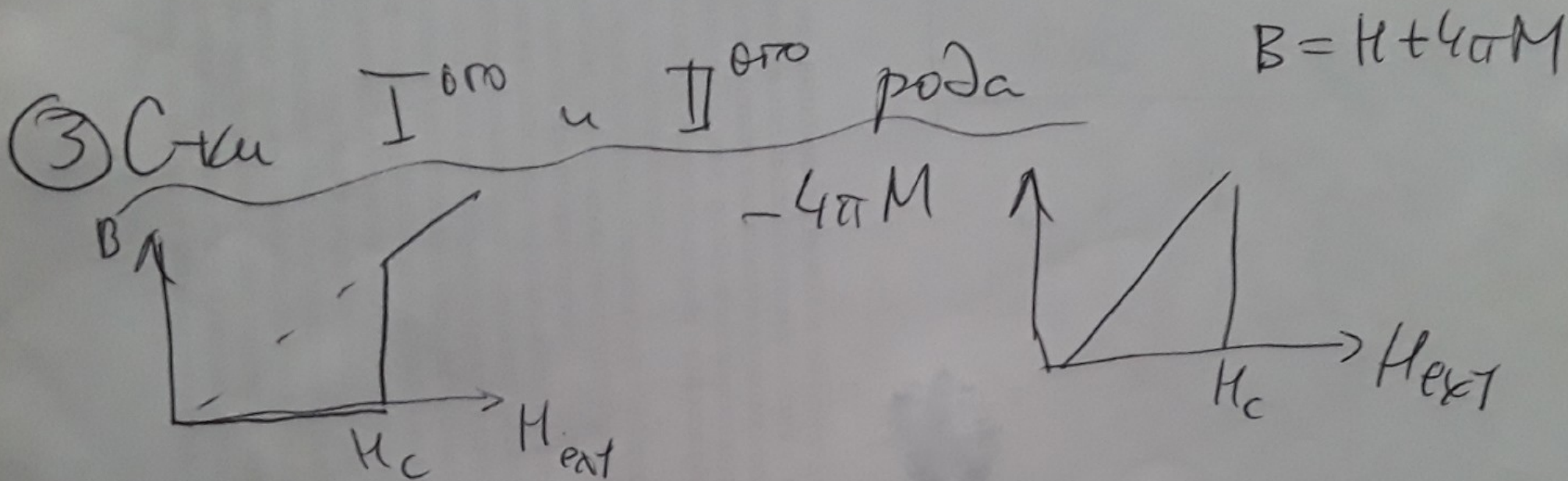
Nearly critical InOx :

B.Sacepe M.Ovadia
D.Shahar (2009)



Часть 2: Основные общие свойства сверхпроводников

Сверхпроводники 1-ого и 2-ого рода



Промежуточный
[см. ЭСС §3, §29]

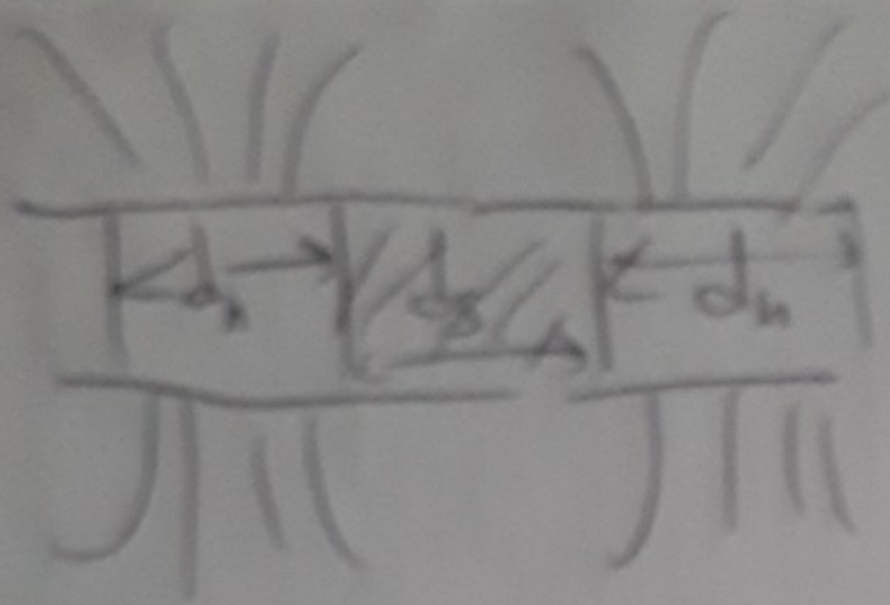
состояние

$$(1 - n)H + nB = H_{ext}$$

$$H = \frac{H_{ext}}{1 - n}$$

$$B_{n1} = B_{n2}$$

$$H_{t1} = H_{t2}$$

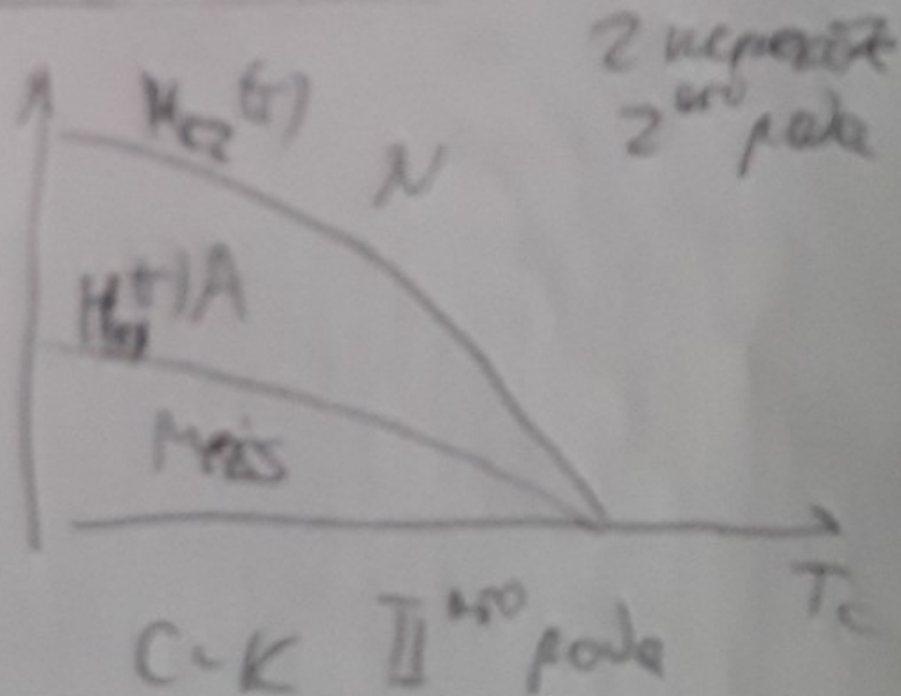
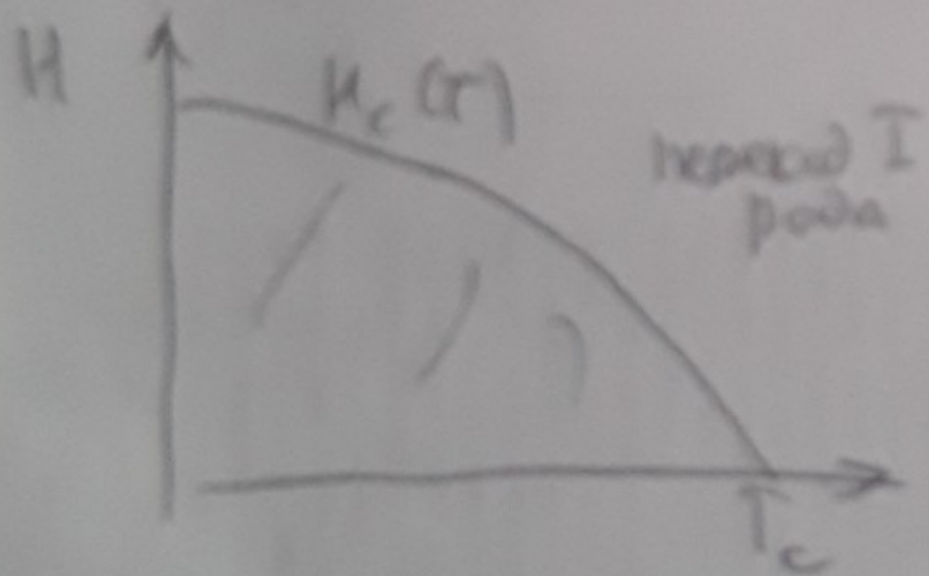


Уравнение
 $SE = N$ ошибки

Уравнение S/N имеет коверку, нет смысла

C-к II рода [предсказан А. Адрикозовичем, 1957]
 ведет себе не так!

Фазовые диаграммы на (H-T):



$\sigma < 0$, поле проникает в виде
вихрей $\Phi_v = \Phi_0 = \frac{2\hbar c}{e}$

Термодинамика с-ков I рода

(5)

$$F_{SH} = F_{S0} + \frac{H^2}{8\pi}$$

$$-M \Delta H = \frac{H \Delta H}{4\pi} = d\left(\frac{H^2}{8\pi}\right)$$

переход при I рода

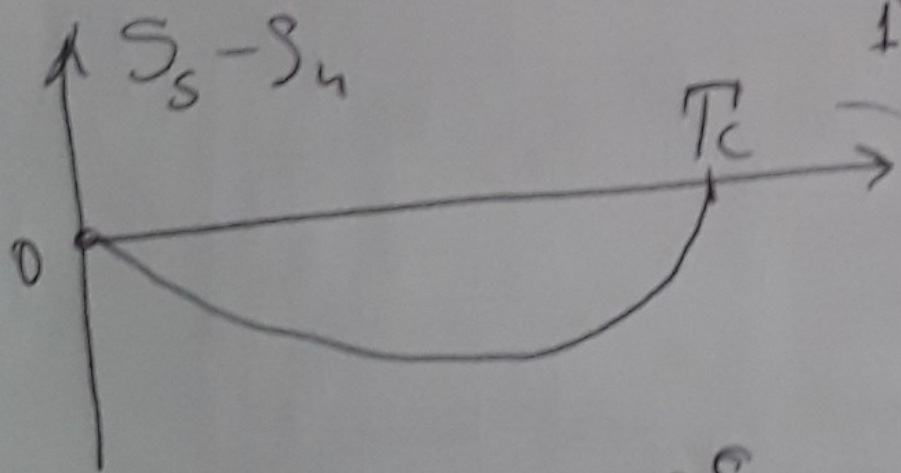
$$F - F_{S0} = \frac{H_{ex}^2}{8\pi}$$

Энтальпия: $S = -\frac{\partial F}{\partial T}$

$$-S_n + S_s = \frac{H_c}{4\pi} \frac{\partial H_c}{\partial T}$$

$$S_c(T) = S_n(T) - \frac{H_c}{4\pi} \left| \frac{\partial H_c}{\partial T} \right|$$

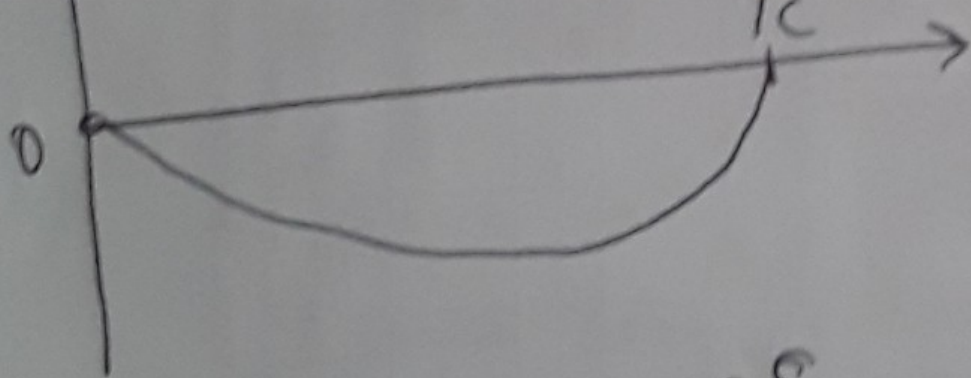
$$\underline{\underline{S_c < S_n}}$$



1) S — упорядоченность

2) $\Delta S = 0$ при $T = T_c$
 $H = 0$

3) $\Delta S \neq 0$ при $H \neq 0$
т.е. переход I рода



3) $\Delta S \neq 0$ при $H \neq 0$
 т.е. непереход 1^{ого} рода

Если T^* короче $\frac{\partial \Delta S}{\partial T} = 0$, т.е.

$$\zeta = C_H$$

$$\zeta - C_H = \frac{T^2 (\zeta - C_H)}{2T} = \frac{T}{4\pi} \left[\left(\frac{\partial H_C}{\partial T} \right)^2 + H_C \frac{\partial^2 H_C}{\partial T^2} \right]$$

$$\Delta C \Big|_{T_c} = \frac{T_c}{4\pi} \left(\frac{\partial H_C}{\partial T} \right)^2 \leftarrow \underline{\text{конечный скачок!}}$$

Делектоини параметри с-св

9

$$\vec{j} = \sigma \vec{E} = \sigma \cdot \frac{1}{c} \dot{\vec{A}}$$

Матрица: $\vec{j}_\omega = \frac{i\omega\sigma}{c} \delta \vec{A}_\omega$

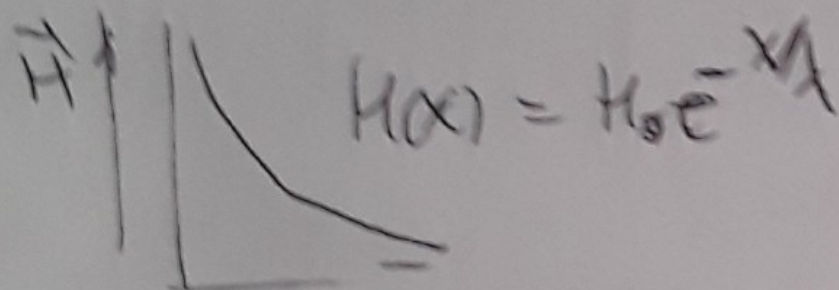
с-к: $\vec{j}_\omega = \frac{i\rho_s c}{\omega} \Rightarrow \vec{j} = -\int \rho_s \delta \vec{A}$
 $= -\frac{c}{4\pi\lambda^2} \delta \vec{A}$

$$\vec{\nabla}_x \vec{H} = \frac{4\pi}{c} \vec{j}$$

$$\vec{\nabla}_x \vec{\nabla}_x \vec{A} = -\frac{1}{\lambda^2} \vec{A}$$

$$\Rightarrow \frac{d^2 A}{dx^2} = \frac{1}{\lambda^2} A$$

$$A \frac{d^2 j}{dx^2} = \frac{1}{\lambda^2} j$$



$$F_{\text{эл}} = F_{\text{ст}} + \frac{1}{8\pi} \int_V \left\{ \vec{H}^2 + \lambda^2 [\vec{\nabla} \times \vec{H}]^2 \right\} d^3x$$

квадратичное Лагранжиан для св. энергии
 $\hbar^2 m_0^2 / 2 = m_0^2 \hbar^2 c^2 = \frac{m_0}{2c^2} \left(\frac{c}{4\pi} [\vec{\nabla} \times \vec{H}] \right)^2 = \frac{\lambda^2}{8\pi} (\vec{\nabla} \times \vec{H})^2$ где $\lambda^2 = \frac{m_0^2 c^2}{4\pi \hbar^2 c^2}$
 Не всегда применимо:

$$\vec{j}^{\text{эл}} = - \int Q(\vec{r}, \vec{r}') \vec{A}(\vec{r}') d\vec{r}'$$

$|\vec{r} - \vec{r}'| \sim \xi_0$ (размер ядра)

если $\xi_0 \gtrsim \lambda$, то

неполнота действия

предел Пуанкаре

$$\lambda = \frac{\int H_0(x) dx}{M_0}$$

повтор!
 беспрерывно

Кинетическая энергия

$$F^M = \frac{1}{8\pi} \int H^2 d^3r = \frac{1}{2c^2} L^M I^2$$

$$F^K = \frac{\lambda^2}{8\pi} \int (\nabla \times \bar{A})^2 d^3r = \frac{1}{2c^2} L^K I^2$$

$$\bar{j}_s = n_s e \bar{v}_s$$

$$L^k = \frac{4\pi \lambda^2}{I^2} \int j_s^2 d^3r$$

нормировка $R \gg \lambda$

$$j_s = j_{s0} e^{-x/\lambda}$$

$$L_{\square}^k = 2\pi \lambda$$

$$I = 2\pi R \lambda j_{s0}$$

$$L^k = L_z \frac{\lambda}{R}$$

$$L_{\square}^M = 2\pi \lambda$$