# Лекция 8 Флуктуации в сверхпроводящих контактах

① $$I = I_c \sin\varphi + \frac{\hbar}{2e}\frac{\dot\varphi}{R} + \underset{\text{флукт.}}{j(t)}$$

$$\frac{\hbar}{2e}\frac{\dot\varphi}{R} = -\frac{2e}{\hbar}\frac{\partial}{\partial\varphi}\underbrace{\left[-E_J\cos\varphi - \frac{\hbar}{2e}I\varphi\right]}_{U_{tot}(\varphi)} + j(t)$$

$$\dot\varphi = -\Gamma\frac{\partial U_{tot}}{\partial\varphi} + \xi(t)$$

$$\langle\xi(t)\,\xi(t')\rangle = 2T\Gamma\,\delta(t-t')$$

$$\Gamma = \left(\frac{2e}{\hbar}\right)^2 R$$

$P(\varphi, t)$

$$\frac{\partial P}{\partial t} \cdot \frac{1}{\Gamma} = \frac{\partial}{\partial \varphi}\left( T \frac{\partial P}{\partial \varphi} + \frac{\partial U_{tot}}{\partial \varphi} P \right)$$

Стационарное $P_{st}(\varphi)$

Равновесное $P_0 = \frac{1}{Z} \exp\left[ - \frac{U_{tot}(\varphi)}{T} \right]$ Гиббс

При $I = 0$ – OK, но при $I \neq 0$ не годится – – периодичность нарушена

Более общее случ: $T\frac{\partial P_{st}}{\partial \varphi} + \frac{\partial U_{tot}}{\partial \varphi} P_{st} = \text{const}$

$T/E_J = \tau \qquad I/I_c = j \qquad I_c = \frac{2e}{\hbar} E_J$

$$\frac{\partial P}{\partial \varphi} + \frac{1}{\tau}(\sin\varphi - j\cancel{\varphi})P = Q$$

$$P_1(\varphi) = C(\varphi)\exp\left[+\frac{1}{\tau}(\cos\varphi + j\varphi)\right]$$

$$\frac{dC}{d\varphi} = Q\exp\left[-\frac{1}{\tau}(\cos\varphi + j\varphi)\right]$$

$$C(\varphi) = Q\int_{\varphi_0}^{\varphi} d\tilde{\varphi}\, \exp\left[-\frac{1}{\tau}\left(\cos\tilde{\varphi} + j\tilde{\varphi}\right)\right]$$

① $\int_0^{2\pi} P_{st}(\varphi)\,d\varphi = 1$ ② $P_{st}(\varphi + 2\pi) = P_{st}(\varphi)$

Эти условия определяют $\varphi_0$ и $Q$

$$\boxed{P_{st}(\varphi) = \exp\left[+\frac{1}{\tau}\left(\cos\varphi + j\varphi\right)\right] Q\int_{\varphi_0}^{\varphi} d\tilde{\varphi}\, \exp\left[-\frac{1}{\tau}\left(\cos\tilde{\varphi} + j\tilde{\varphi}\right)\right]}$$

Среднее напряжение $\overline{V} = \frac{\hbar}{2e}\overline{\dot{\varphi}} = \overline{R\left(I - I_c \sin\varphi\right)} =$

$$= \frac{\hbar}{2e}\cdot\Gamma(-)\overline{\frac{\partial U_{tot}}{\partial\varphi}} =$$

$$= R I_c \int_0^{2\pi} d\varphi \left(j - \sin\varphi\right) P_{st}(\varphi)$$

# Квантовые флуктуации и распад

Полная энергия Дж. контакта

$$E = \frac{C}{2}V^2 - E_J \cos\varphi$$

Лагранжиан: $$\mathcal{L} = \frac{C}{2}\left(\frac{\hbar}{2e}\dot{\varphi}\right)^2 + E_J \cos\varphi$$

$$P = \frac{\partial L}{\partial \dot{\varphi}} = C\left(\frac{\hbar}{2e}\right)^2 \dot{\varphi} \qquad \dot{\varphi} = \left(\frac{2e}{\hbar}\right)^2 C^{-1} P$$

Гамильтониан $$\dot{\varphi}\frac{\partial L}{\partial \dot{\varphi}} - L = H = \frac{(2e)^2}{2C}\left(\frac{P}{\hbar}\right)^2 - E_J \cos\varphi$$

$$P = C\left(\frac{\hbar}{2e}\dot{\varphi}\right)\cdot\frac{\hbar}{2e} = CV\frac{\hbar}{2e} = \frac{Q}{2e}\cdot\hbar$$

Т.е. $\hat{H} = \frac{Q^2}{2C} - E_J\cos\varphi$, причем

$$[\hat{Q}, \hat{\varphi}] = -i(2e) \quad \hat{Q} = \frac{1}{i}\frac{\partial}{\partial\varphi}(2e)$$

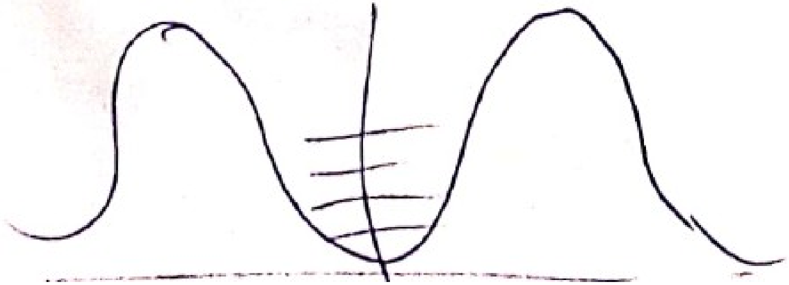
Зарядовая энергия $4E_c\hat{N}^2$ где $\hat{N} = \frac{\hat{Q}}{2e}$

$$E_c = \frac{e^2}{2C}$$

$$[\hat{N}, \hat{\varphi}] = -i$$

. Квазикласс. прибл. для контакта.

$E_J \gg E_C$



Число уровней в яме:

$n_{max} \sim (E_J/8E_C)^{1/2}$

) осцилляторные уровни

$$\hat{H} = + 4E_C\hat{N}^2 + \frac{E_J}{2}\hat{\varphi}^2 + \mathrm{const}$$

$$\omega_J^2 = 8E_CE_J \qquad \frac{\omega_J}{E_J} = \sqrt{\frac{8E_C}{E_J}}$$

б) Ток $I \simeq I_c$ :

$$-E_J \cos\varphi - \frac{\hbar}{2e} I \varphi = E_{tot}$$


$$\sin\varphi_* = \frac{I}{I_c} = j \lesssim 1$$

$$\varphi_* = \frac{\pi}{2} \pm \psi \qquad \cos\psi = j \qquad \frac{\psi^2}{2} = 1 - j$$

$$\psi_\pm = \pm\sqrt{2(1-j)}$$

$$\left.\frac{\partial^2 U}{\partial \varphi^2}\right|_{\varphi_*} = E_J \cos\varphi_- = E_J \cdot \sqrt{2(1-j)}$$

$$\Delta U = \frac{2}{3} E_J \sqrt{2} (1-j)^{3/2}$$

Частота осцилляторов... $\sqrt{2(1-j)}\,E_J$ вместо $E_J$

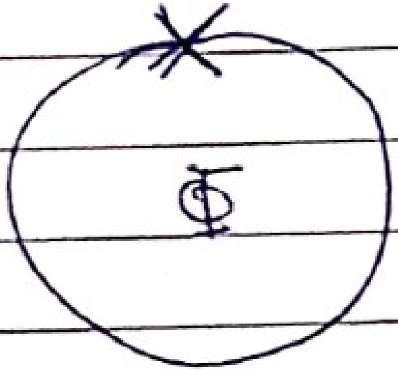
$$\omega_J^2(j) = 8E_c E_J \sqrt{2}(1-j)^{1/2}$$

$$\omega_J \sim (1-j)^{1/4} \qquad \frac{\Delta U}{\omega_J} \sim \frac{E_J}{E_c}(1-j)^{5/4}$$

Вероятность распада в ед. вр.:

$$P \simeq \omega_J(j)\exp\left[-\frac{S_{tun}}{\hbar}\right]$$

Квант. флукт.: TLS



$$U_{пот}(\varphi) = -E_J\cos\varphi + \frac{LI^2}{2c^2}$$

$$I = I_c \sin\varphi\ ; \quad \varphi = \frac{2\pi\Phi}{\Phi_0}$$

$$\Phi = \Phi_{ext} - \frac{LI}{c}$$

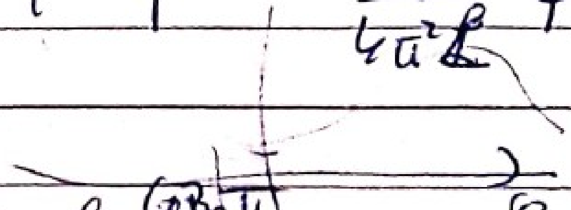
$$U_{tot}(\varphi) = -E_J\cos\varphi + \frac{\mathcal{L}}{2C^2}\frac{C^2}{\mathcal{L}^2}\cdot\frac{\Phi_0^2}{(2\pi)^2}\left(\frac{2\pi\Phi_{ext}}{\Phi_0} - \varphi\right)^2$$

$$= -E_J\cos\varphi + \frac{\Phi_0^2}{8\pi^2\mathcal{L}}(\alpha-\varphi)^2$$

a) $\alpha = \pi$ $\quad \frac{\partial U_{tot}}{\partial\varphi} = 0 \Rightarrow E_J\sin\varphi - \frac{\Phi_0^2}{4\pi^2\mathcal{L}}(\pi-\varphi) = 0$

$\pi-\varphi = \psi \quad \frac{\Phi_0^2}{4\pi^2\mathcal{L}}\psi = E_J\sin\psi \quad \psi = \beta_L\sin\psi$

$\beta_L < 1 \Rightarrow \psi = 0$



$\beta_L > 1 \Rightarrow \psi_1 = 0$

$\psi_{2,3} = \pm\psi_*$

$$\frac{\frac{\hbar}{2e}I_c\mathcal{L}\cdot 4\pi^2}{2\pi\,\hbar c/2e\,\Phi_0} = \frac{2\pi\mathcal{L}I_c}{c\,\Phi_0}$$