Sachdev-Ye-Kitaev Model with Quadratic Perturbations: The Route to a Non-Fermi Liquid

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We study stability of the Sachdev-Ye-Kitaev (SYK_4) model with a large but finite number of fermions N with respect to a perturbation, quadratic in fermionic operators. We develop analytic perturbation theory in the amplitude of the SYK_2 perturbation and demonstrate stability of the SYK_4 infrared asymptotic behavior characterized by a Green function $G(\tau) \propto 1/\tau^{3/2}$, with respect to weak perturbation. This result is supported by exact numerical diagonalization. Our results open the way to build a theory of non-Fermi-liquid states of strongly interacting fermions.

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The wealth of available data on various strongly correlated electronic materials [1,2] calls for the development of a general theory of the non-Fermi-liquid ground state(s) of an interacting many-body fermionic system. Still, no general theoretical scheme leading to such a behavior in the zero-temperature limit is known (for a recent review see Ref. [3]). Mathematically, the complexity of the problem is due to the absence of any general method to calculate non-Gaussian functional integrals that appear in the theory of strongly interacting fermions.

A new and fresh view on this old problem is provided by the recently proposed [4–6] Sachdev-Ye-Kitaev (SYK) model of interacting fermions. It has attracted a lot of attention recently as a possible boundary theory of a two-dimensional gravitational bulk [5,7,8]. The original SYK model contains $N \gg 1$ Majorana fermions, with the Hamiltonian consisting of a sum of all possible 4-fermion terms with random matrix elements $J_{ijkl} \sim J/N^{3/2}$ [note that the free (quadratic) term is missing in the SYK Hamiltonian]. This model can be considered a nonlinear generalization of usual random-matrix Hamiltonians [9]. Furthermore, SYK_q models with arbitrary $q = 2k$, were introduced and studied [8]. These models provide the most straightforward way to enhance the role of interaction between fermions, avoiding formation of any simply ordered structures that lead—usually, but not always [3]—to a breakdown of some evident symmetry of the Hamiltonian.

The SYK model is analytically tractable in the large-N limit and shows two different types of asymptotic behavior for the fermionic Green function $G(\tau)$. In the intermediate time range $1/J \ll \tau \ll t_c$, with $t_c \sim N/J$, the self-consistent approximation for interaction self-energy is valid and $G(\tau) \propto \tau^{-1/2}$. For even larger times $\tau \gg t_c$, it was found in Ref. [10] that fluctuations, beyond the self-consistent treatment, change the behavior of the Green function to $G(\tau) \propto \tau^{-3/2}$ (we treat exponentially large ergodic timescale $\propto 2^{N/2}$ as being infinite). Both these types of behavior are crucially different from the standard Fermi-liquid scaling $G(\tau) \propto 1/\tau$, which corresponds to the nonzero finite density of low-energy states. In other terms, low-energy excitations of the SYK model are not described by any kind of quasiparticles.

For the reasons described above, the SYK model seems to be a very promising starting point to approach a theory of non-Fermi-liquid ground state. A few problems arise, however: (i) the absence of a quadratic term in the Hamiltonian makes pure SYK Hamiltonian unrealistic for electronic systems; (ii) the original SYK model contains Majorana fermions, which are quite scarce in nature (see, however, the few relevant proposals in Refs. [11–13]); and (iii) the most interesting properties of a non-Fermi-liquid state are those related to transport phenomena, while SYK is a random-matrix-type model without spatial coordinates. Quite a number of recent publications address the issues listed above [14–17]. Phase transitions controlled by the ratio of numbers of fermions in two SYK-like subsystems were studied in Refs. [14,18].

The generalization of the SYK model for complex fermions was developed in Refs. [15,17]. A sequence of SYK "quantum dots" connected by weak (quadratic) tunnelling was considered in Refs. [16,17], making it possible to define and study transport quantities like resistance, thermal resistance, etc.; see also a very recent, extensive study in the same direction [19]. A somewhat different direction was explored in Refs. [20,21], where a "dispersive SYK model" was introduced and studied in a way similar to Ref. [17].

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All these studies are restricted by the use of self-consistent ($N \to \infty$) approximation, with the exception provided by Ref. [22], where some renormalization group procedure based upon expansion over small $q - 2 \ll 1$ was formulated. Numerical analysis of the finite-range generalization of the SYK model with quadratic terms was performed in Ref. [23], and analogies with many-body localization were discussed.

However, all (known to us) studies of stability of SYK behavior with respect to quadratic perturbations, indicate its runaway instability. As it was shown in Refs. [14,16,17,19] in the framework of the self-consistent approximation, the scaling dimension of the SYK_2 perturbation is negative when estimated within the conformal limit, corresponding to the timescales $1/J \ll \tau \ll t_c$. References [17,19] demonstrate an interesting non-Fermi-liquid behavior in the intermediate temperature region $T^* < T \ll J$, but still obtain Fermi-liquid behavior in the lowest T range below T^*.

In the present Letter we reconsider the problem of the SYK_4 stability with respect to quadratic perturbations, going beyond the saddle-point approximation. We study fermionic Green function in the region $\tau \gg t_c$ by means of perturbation theory in the amplitude of SYK_2 terms, using the infrared asymptotic solution [8,10] as a starting point. We show analytically that a weak SYK_2 perturbation does not change the $G(\tau) \propto 1/\tau^{3/2}$ asymptotics of the Green function, but simply renormalizes the coefficient. This result proves the existence of a *domain of stability*, with a nonzero area in the parameter space of Hamiltonians, where a non-Fermi liquid is realized as a ground state. We also performed a numerical analysis of the Green function of the mixed $SYK_4 + SYK_2$ model to support our analytic study.

The model and basic equations.—We consider the model defined by the following Hamiltonian:

$$H = \frac{1}{4!}\sum_{i,j,k,l} J_{i,j,k,l}\chi_i\chi_j\chi_k\chi_l + \frac{i}{2!}\sum \Gamma_{i,j}\chi_i\chi_j \tag{1}$$

where χ_i are Majorana fermions and all indices run from 1 to N. The matrix elements $J_{i,j,k,l}$ and $\Gamma_{i,j}$ are fully antisymmetric, and independent random Gaussian variables with zero mean and the variances $\langle J^2_{i,j,k,l}\rangle = (3!J^2/N^3)$, $\langle \Gamma^2_{i,j}\rangle = (\Gamma^2/N)$. The functional integral representation of this theory is described by the action $S = -(N/2)(S_1 + S_2)$ with two contributions [8,10]:

$$S_1 = \mathrm{tr}\log(\partial_\tau - \Sigma_{\tau\tau'}) + \int d\tau d\tau' \left(\frac{J^2}{4}G^4_{\tau\tau'} - \Sigma_{\tau\tau'}G_{\tau'\tau}\right),$$

and

$$S_2 = \int d\tau d\tau' \frac{\Gamma^2}{2}G^2_{\tau\tau'}. \tag{2}$$

In the limit $N \gg 1$, the mean-field analysis is appropriate and the corresponding saddle-point equations read as follows:

$$\partial_\tau G_{\tau\tau'} - \int d\tau'' \Sigma_{\tau\tau''}G_{\tau''\tau'} = \delta(\tau - \tau'), \tag{3}$$

$$\Sigma_{\tau\tau'} = J^2 G^3_{\tau\tau'} + \Gamma^2 G_{\tau\tau'}. \tag{4}$$

We are going to study corrections to the SYK model Green function $G(\tau)$, assuming the dimensionless parameter $\gamma = \Gamma/J$ to be small. Within applicability range of the saddle-point Eqs. (3) and (4), the scaling dimension of the perturbation is negative, $\Delta_\gamma = -\frac{1}{2}$. As a result, $G^{(0)}_{\mathrm{SYK}}(\tau) \propto (J\tau)^{-1/2}$ (the mean-field solution at $\gamma = 0$) is unstable with respect to the perturbation: at $\tau \geq \tau^* \sim 1/J\gamma^2$ it is replaced by the usual Fermi-liquid behavior $G(\tau) \propto (J\gamma\tau)^{-1}$. On the other hand, at sufficiently long times $t \gg t_c$, soft reparametrization modes [5,6,8] become relevant and the Green function of the pure SYK_4 model acquires different scaling [10]

$$G(\tau) = \frac{1}{(4\pi)^{1/4}}\frac{1}{\sqrt{Jt_c}}\left(\frac{t_c}{\tau}\right)^{\frac{3}{2}} \equiv \frac{\Gamma^4(\frac{1}{4})}{\sqrt{2MJ}\pi^{5/4}}\left(\frac{M}{\tau}\right)^{\frac{3}{2}}. \tag{5}$$

For the reasons that become clear soon, we introduced a new notation, $M = \pi t_c/\Gamma^4(\frac{1}{4})$, where $\Gamma(x)$ is the Euler gamma function.

For a sufficiently weak perturbation $\gamma \ll 1/\sqrt{N}$, the crossover timescale τ^* becomes larger than t_c and loses its relevance: the analysis of the SYK solution stability should now be performed using the asymptotic behavior [Eq. (5)] as a starting point. Before we develop this analysis, a brief reminder on the origin of the result [Eq. (5)] is in order.

The saddle-point solution of Eqs. (3) and (4) at $\Gamma = 0$ is invariant with respect to the reparametrization of time, $\tau \to f(\tau)$, which is an approximate symmetry of the full action S_1, see Refs. [4–6]. Fluctuations around the saddle point can be accounted for by a kind of "sigma-model" defined on the manifold of functions $\phi(\tau)$, defined via the relation $df/dt = e^{\phi(t)}$. This field theory has a very simple action [10]:

$$S_\phi = \frac{M}{2}\int (\phi')^2 d\tau. \tag{6}$$

The asymptotic behavior [Eq. (5)] of the Fermionic Green function $G_{\tau\tau'} = (1/N)\sum_i \chi_i(\tau)\chi_i(\tau')$ can then be obtained by the averaging of the functional

$$G_{\tau\tau'}[\phi(\tau)] = \frac{1}{\sqrt{2J\sqrt{\pi}}}\mathrm{sgn}(\tau - \tau')\frac{e^{\phi(\tau)/4}e^{\phi(\tau')/4}}{|\int_{\tau'}^{\tau} e^{\phi(\tilde{\tau})}d\tilde{\tau}|^{1/2}}, \tag{7}$$

with the action [Eq. (6)] being applicable in the long time limit $\tau \gg t_c$. For actual calculations of functional integrals like Eq. (7) with the action [Eq. (6)], we follow

Refs. [10,24] where a very useful reduction to the Liouville quantum mechanics [25–28] was employed (see also Refs. [29–31]).

There are various results in the literature [8,10] concerning the determination of the important parameter $M = M(N, J)$ which enters the action [Eq. (6)]. We prefer to employ the relation between results for the full density of states of the SYK_4 model obtained (i) via an asymptotic low-energy theory, expressed in terms of M: $\rho(\epsilon) \propto \sinh(2\pi\sqrt{2M\epsilon})$ [24], and (ii) by the method of generalized orthogonal polynomials [32]: $\rho(\epsilon) \propto \sinh[(2\pi\sqrt{2}\sqrt{\epsilon/\epsilon_N})/(\ln 1/\eta_N)]$, where $\eta_N = 1-(32/N)+O(1/N^2)$ and $\bar{\epsilon}_N = (JN/16\sqrt{2}) + O(1)$ (at a finite N, the expressions are given in Ref. [32]). Comparison of two approaches yields

$$M = \frac{m(N)}{32\sqrt{2}}\frac{N}{J}, \tag{8}$$

where interpolating function $m(N)$ approaches 1 in the limit $N = \infty$. Note that convergence of $m(N)$ upon the increase of N is very slow; in particular, $m(32) \approx 0.54$. Note also the numerical factor ~0.02 in the RHS of Eq. (8), which makes M much smaller than N/J. Fortunately, the actual timescale that enters Green functions $G(\tau)$ is $t_c = M\Gamma^4(\frac{1}{4})/\pi \approx 55M \approx 1.2Nm(N)/J$; it will be important below for the comparison with numerical data at a large but finite $N = 32$.

Perturbation theory.—First-order correction to the Green function $G(\tau)$ due to the quadractic term S_2 in the action can be found (see the Supplemental Material [33] for more details) in a straightforward way as follows [notation $\langle .. \rangle_0$ means the average over ϕ field with the action S_ϕ, see Eq. (6)]:

$$\delta G(\tau) = -\langle G_{\tau,0}[\phi]S_2[\phi]\rangle_0 + \langle G_{\tau,0}\rangle_0\langle S_2\rangle_0 \tag{9}$$

Substituting here Eq. (2), we find that the first term of Eq. (9) contains an average [over $\phi(\tau)$ fluctuations] of the product of three functionals like Eq. (7), with the time arguments $0, \tau$, and τ_1, τ_2, where further integration over $\tau_{1,2}$ is implied. Functional integration over $\phi(\tau)$ with the action [Eq. (6)] should be performed separately in six different time regions with the following order of time arguments:

1	2	3	4	5	6
$\tau_2, \tau_1, 0, \tau$	$\tau_2, 0, \tau_1, \tau$	$\tau_2, 0, \tau, \tau_1$	$0, \tau_2, \tau_1, \tau$	$0, \tau_2, \tau, \tau_1$	$0, \tau, \tau_2, \tau_1$

Domains 1 and 6 have a trivial structure and their contributions are canceled completely by the second term in Eq. (9). Combining other contributions with the corresponding parts of the second term in Eq. (9), we find

$$\delta G(\tau) = \frac{N\sqrt{MJ}}{4\pi^{5/4}}\gamma^2\left[\sum_{i=2}^{5} f_i\left(\frac{\tau}{2M}\right) - f_Z\left(\frac{\tau}{2M}\right)\right], \tag{10}$$

where functions $f_i(x)$ (for $i = 2, 3, 4, 5$) and $f_Z(x)$ are defined and calculated in the Supplemental Material [33]. In total, in the long-time limit $\tau \gg M$ we have the following:

$$\delta G(\tau) = cN\sqrt{MJ}\gamma^2(\tau/M)^{-\frac{3}{2}}, \tag{11}$$

with $c \approx 108$. Comparing with Eq. (5), we find $\delta G/G \approx 3.7JMN\gamma^2$. As a result

$$\delta G/G \approx 0.081m(N)N^2\gamma^2, \tag{12}$$

recalling that $m(N)$ is defined in the Eq. (8). Equation (12) demonstrates that relevant parameter of the perturbation theory in the infrared limit is actually $\gamma N \equiv b$, and perturbation of the SYK_2 type only modifies the numerical prefactor in $G(\tau)$.

Numerical data.—Below, we present numerical results for the Green function $G(\epsilon)$ of the $SYK_2 + SYK_4$ model in the energy representation. This Green function can be numerically studied with two complementary approaches: via exact diagonalization at a finite N and directly in the limit of $N \to \infty$ via the solution of the mean-field equations [Eqs. (3) and (4)]. Within the numerical analysis below, we put $J = 1$. For exact diagonalization, we consider the Hamiltonian of Eq. (1) for $N = 32$ fermions for the range of γ and average over hundreds of disorder realizations (we employ the representation of Majorana algebra, used in Ref. [34]). This results into red, blue, and brown curves in Fig. 1. The most interesting regime is realized at small $\epsilon \leq 1/t_c$ [that is, below the maximum of the function $G(\epsilon)$]. The characteristic timescale $t_c \approx 20$ for $N = 32$. Unfortunately, the region $\epsilon \leq 1/t_c$ is limited from below by the many-body energy scale $\epsilon = \epsilon_{MB}$ (defined as an energy, counted from the ground state, where the many-body level spacing $\delta\epsilon(\epsilon)$ becomes comparable to ϵ itself.) For $N = 32$ the corresponding cutoff $\epsilon_{MB} \approx 2 \times 10^{-3}$ determines the left edge on Fig. 1. The respective energy interval $\epsilon_{MB} \ll \epsilon \ll M^{-1}$ is not really large enough to admit the predicted asymptotic behavior $G(\epsilon) \propto \epsilon^{1/2}$: compare with the dotted line, which is evaluated according to the theoretical prediction of Eq. (22) of Ref. [10]. However, qualitatively at $\gamma = 0$, we find the expected behavior. At a small $\gamma \ll N^{-1}$ the corresponding part of $G(\epsilon)$ dependence shifts up in the log-log plot (see blue curve for $\gamma = 0.01 \ll 1/N$) without a change of behavior as a function of ϵ, in agreement with our analytical result [Eq. (12)]. However, at slightly larger values of $\gamma \sim 1/N$ (brown curve) the Green function $G(\epsilon)$ saturates at low energies, in agreement with the Fermi-liquid behavior.

At even larger values of $\gamma \geq 1/\sqrt{N}$, the asymptotic region with $G(\epsilon) \propto \sqrt{\epsilon}$ disappears completely, and the Green function can be approximated by the solution of mean-field Eqs. (3) and (4). Interaction term is then

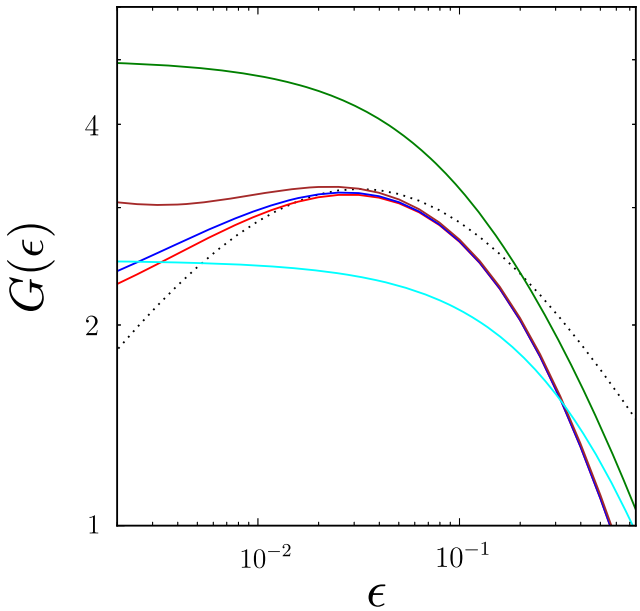

FIG. 1. Green function of the SYK model (log-log scale): (i) red, blue, brown: exact diagonalization for $N = 32$ fermions and $\gamma = 0$, 0.01, 0.03; (ii) green, cyan: solution of mean-field equations Eqs. (3) and (4) at $\gamma = 0.2$ and $\gamma = 0.4$; (iii) dotted line: analytical result of Eq. (22) from Ref. [10], interpolating between $\epsilon \gg M^{-1}$ and $\epsilon \ll M^{-1}$ limits, evaluated at $N = 32$.

important at higher energies $\epsilon \geq \epsilon_\gamma = \gamma^2 J > 1/t_c$ only. The green and cyan curves show the numerical solutions to these equations. Note that mean-field solutions differ in the region $\epsilon \gg 1/t_c$ from the analytical result given by Eq. (22) of Ref. [10], evaluated at $N = 32$ (dotted line in Fig. 1). These deviations appear since the asymptotic region $1/t_c = 0.05 \ll \epsilon \ll 1$ is apparently not wide enough. Thus, finite-size effects are detrimental for all analytically available asymptotics in this problem even for a relatively large system of $N = 32$ fermions. Although available system size is on the border line of emergence of respective asymptotic regions, we believe that the results of ED and mean-field studies are consistent with our analytical estimates. In particular, the low-ϵ limits of $G(\epsilon)$ demonstrated by cyan and green lines are in agreement with analytic result for pure SYK_2 theory, $G(\epsilon \to 0) = 1/\Gamma$.

Conclusions.—Schematically, our results for the zero-temperature phase diagram of the combined SYK_4-SYK_2 model are shown in Fig. 2. We emphasize the somewhat unusual scaling limit of a large N that is employed here. Namely, we consider $N \gg 1$ as some finite number, but we neglect exponentially small many-body level spacing $\epsilon_{MB} \sim 2^{-N/2}$. Then our results demonstrate the presence of a *phase transition* between a fully chaotic non-Fermi-liquid ground state realized at $b \equiv \gamma N < b_c$, and a Fermi-liquid ground state existing at $b > b_c$, with $b_c \sim 1$. We emphasize that the corresponding critical value of the amplitude of the quadratic perturbation Γ equals $\Gamma_c = b_c J/N$. In other terms, the effect of this perturbation

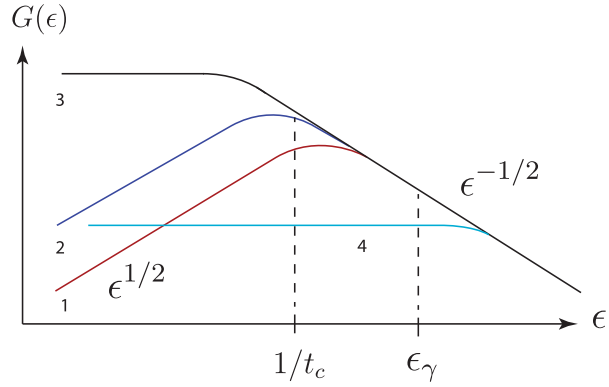

FIG. 2. Sketch of the Green function $G(\epsilon)$ in log-log scale, in the limit of $N \gg 1$ for several values of $b = \gamma N$ ordered as $b_1 < b_2 < b_c < b_3 < b_4$, with critical b_c of the order unity. Transition to Fermi liquid occurs between blue (2) and grey (3) lines. Light-blue line (4) corresponds to large $b \geq \sqrt{N}$ when the 4-fermion interaction is relevant at high energies above $\epsilon_\gamma \gg 1/t_c$ only.

in the infrared limit is much stronger than one could naïvely expect considering its effect at short times $t \leq t_c$ where the relevant Γ scales are $1/\sqrt{N}$.

Note that the $1/t^{3/2}$ long-time asymptotics of the Green function in a pure SYK model can be understood [35] as a result of the square-root edge singularity of the full many-body DOS [24,32], together with the chaotic, nonstructured nature of matrix elements that enter the Lehman expansion for the Green function. Then, the phase transition we found upon an increase of quadratic perturbation b can be understood as a transition to a nonchaotic state, with matrix elements acquiring a nontrivial structure leading to a Fermi-liquid type of behavior $G(t) \sim 1/t$.

Recently, the chaotic-integrable transition for the SYK model with a quadratic perturbation was studied in Ref. [36]. The authors have shown (judging it by the Lyapunov exponent of the 4-point, out-of-time-order correlation function) that for several values of $\gamma \sim O(N^0)$, there exists a finite positive temperature $T = T_c(\gamma)$ such that, at $T > T_c(\gamma)$, the system behaves chaotically, while, at lower temperatures, the Lyapunov exponent drops to zero as it is expected for the Fermi-liquid state with a quasi-particle-based classification of eigenstates. We believe that this transition is of the same kind as we found at zero temperature for a small $\gamma \sim b/N$. In this respect, see a recent study of energy-resolved spectral and many-body-wave-function statistics, reported in Ref. [37]. In particular, Fig. 13(a) [37] demonstrates a qualitative change in the distribution function of the structural entropy of exact eigenstates already between $\kappa = 0$ and $\kappa = 1$ at a low total energy, indicating the phase transition happening at $\kappa \lesssim 1$ in agreement with our results.

It would be very interesting to study a similar Non-Fermi-Liquid–Fermi-Liquid transition in a chain (or lattice) of SYK-like "quantum dots." Note that for such extended

models there is no issue with finiteness of the nonzero many-body level spacing ϵ_{MB} and the problem of the NFL-FL phase transition can be formulated in the strict sense. As was already mentioned in the Introduction, the transition of that kind was studied in Refs. [17,19] at nonzero temperatures within a $N \to \infty$ limit. Another approach was developed in Ref. [38] where an effect of the SYK-like interaction upon the properties of a random-hopping chain was investigated via numerical analysis of the level statistics. The authors of Ref. [38] found the MBL-type transition between fully localized and ergodic ground states at a rather low and decreasing with N ratio J/Γ of the SYK coupling to quadratic coupling; thus it seems to be qualitatively different from the transition we found for a single SYK system.

Finally, we would like to mention an interesting physical problem that may bear some resemblance with the SYK_4-SYK_2 model considered here. It is well known that strong potential disorder suppresses a superconducting transition with nontrivial (d-wave or p-wave) pairing, due to the random mixing of electron states between different sectors of the Fermi surface. However, the mean-square magnitude of the (random-sign) Cooper interaction amplitude survives impurity scattering. It means that electron states with energies exactly at the Fermi surface constitute a kind of SYK-type model with a random 4-fermion interaction. Potentially, this interaction may occur to be strong enough to lead to a non-Fermi-liquid ground state without the formation of any order parameter.

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[1] G. Stewart, Rev. Mod. Phys. **73**, 797 (2001).
[2] G. Stewart, Rev. Mod. Phys. **78**, 743 (2006).
[3] S.-S. Lee, Annu. Rev. Condens. Matter Phys. **9**, 227 (2018).
[4] S. Sachdev, Phys. Rev. X **5**, 041025 (2015).
[5] Talks at KITP on April 7th and May 27th (2015), http://online.kitp.ucsb.edu/online/entangled15/kitaev/, http://online.kitp.ucsb.edu/online/entangled15/kitaev2/.
[6] A. Kitaev and S. J. Suh, J. High Energy Phys. 05 (2018) 183.
[7] J. Polchinski and V. Rosenhaus, J. High Energy Phys. 04 (2016) 001.
[8] J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016).
[9] T. Guhr, A. Müller-Groeling, and H. A. Weidenmüller, Phys. Rep. **299**, 189 (1998).
[10] D. Bagrets, A. Altland, and A. Kamenev, Nucl. Phys. **B911**, 191 (2016).
[11] A. Chew, A. Essin, and J. Alicea, Phys. Rev. B **96**, 121119 (2017).
[12] D. I. Pikulin and M. Franz, Phys. Rev. X **7**, 031006 (2017).
[13] A. Chen, R. Ilan, F. de Juan, D. Pikulin, and M. Franz, Phys. Rev. Lett. **121**, 036403 (2018).
[14] S. Banerjee and E. Altman, Phys. Rev. B **95**, 134302 (2017).
[15] R. A. Davison, W. Fu, A. Georges, Y. Gu, K. Jensen, and S. Sachdev, Phys. Rev. B **95**, 155131 (2017).
[16] S.-K. Jian and H. Yao, Phys. Rev. Lett. **119**, 206602 (2017).
[17] X.-Y. Song, C.-M. Jian, and L. Balents, Phys. Rev. Lett. **119**, 216601 (2017).
[18] X. Chen, R. Fan, Y. Chen, H. Zhai, and P. Zhang, Phys. Rev. Lett. **119**, 207603 (2017).
[19] D. Chowdhury, Y. Werman, E. Berg, and T. Senthil, Phys. Rev. X **8**, 031024 (2018).
[20] P. Zhang, Phys. Rev. B **96**, 205138 (2017).
[21] A. Haldar, S. Banerjee, and V. B. Shenoy, Phys. Rev. B **97**, 241106 (2018).
[22] D. Ben-Zion and J. McGreevy, Phys. Rev. B **97**, 155117 (2018).
[23] A. M. García-García and M. Tezuka, arXiv:1801.03204.
[24] D. Bagrets, A. Altland, and A. Kamenev, Nucl. Phys. **B921**, 727 (2017).
[25] A. Zamolodchikov and A. Zamolodchikov, Nucl. Phys. **B477**, 577 (1996).
[26] D. G. Shelton and A. M. Tsvelik, Phys. Rev. B **57**, 14242 (1998).
[27] J. Teschner, Classical Quantum Gravity **18**, R153 (2001).
[28] Y. Nakayama, Int. J. Mod. Phys. A **19**, 2771 (2004).
[29] T. G. Mertens, G. J. Turiaci, and H. L. Verlinde, J. High Energy Phys. 08 (2017) 136.
[30] D. Stanford and E. Witten, J. High Energy Phys. 10 (2017) 008.
[31] A. Kitaev and S. J. Suh, arXiv:1808.07032.
[32] A. M. García-García and J. J. Verbaarschot, Phys. Rev. D **96**, 066012 (2017).
[33] Please see the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.121.236601 for explicit calculation of the first-order correction.
[34] M. Haque and P. McClarty, arXiv:1711.02360.
[35] We thank the referee of this Letter for mentioning to us this relation.
[36] A. M. García-García, B. Loureiro, A. Romero-Bermúdez, and M. Tezuka, Phys. Rev. Lett. **120**, 241603 (2018).
[37] T. Nosaka, D. Rosa, and J. Yoon, J. High Energy Phys. 09 (2018) 041.
[38] X. Dai, S.-K. Jian, and H. Yao, arXiv:1802.10029.