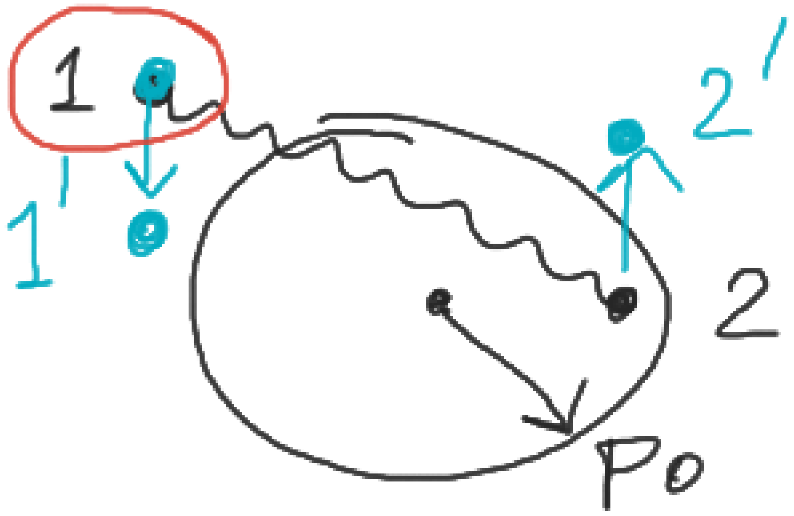
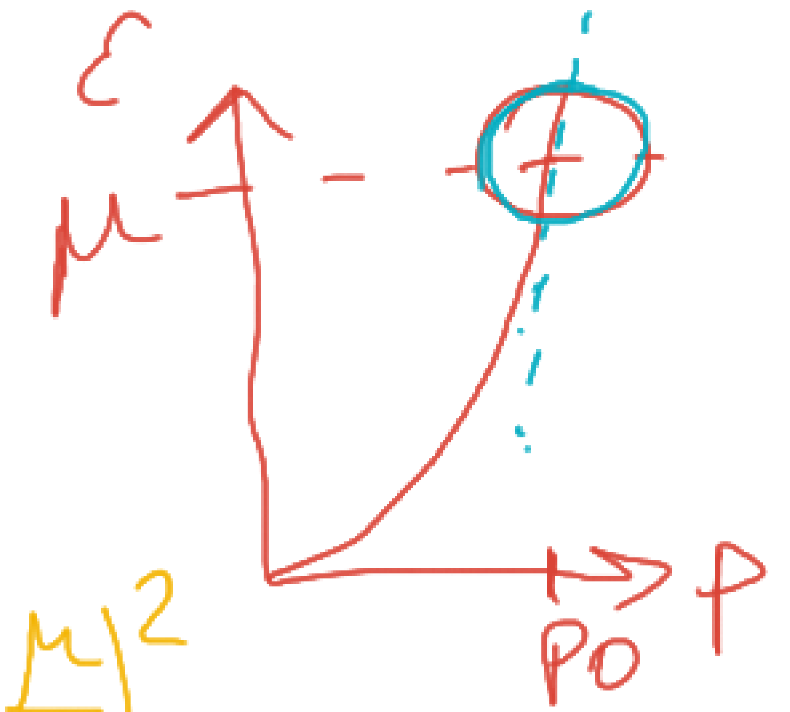
VI. e-e рассеяние



$$\vec p_1 + \vec p_2 = \vec p_1{}' + \vec p_2{}'$$

$p_1,\ p_1',\ p_2' > p_0$, $p_2 < p_0$

$$\xi_1 = \varepsilon(p_1) - \mu = \frac{p_1^2}{2m} - \frac{p_0^2}{2m} = \frac{(p_1+p_0)(p_1-p_0)}{2m} \approx v\,(p_1-p_0)$$



$$\frac{1}{\tau} \propto \int |V|^2\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\,d^3\vec p_2\,d^3\vec p_1{}'$$

$$\frac{1}{\tau} \propto \xi_1^2 \qquad \xi \sim T \qquad \frac{\hbar}{\tau} \sim \frac{T^2}{\mu}$$

$$\boxed{\tau \sim \hbar\frac{\mu}{T^2}}$$

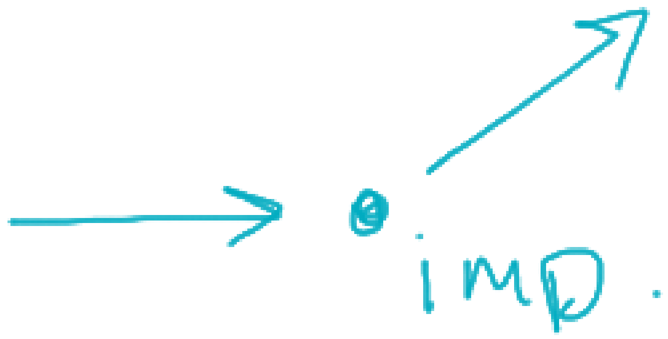
$$\sigma = \frac{n_e e^2 \tau}{m} \sim \frac{n_e e^2 \hbar \mu}{m T^2} \sim \frac{\mu}{\hbar}\left(\frac{\mu}{T}\right)^2$$

$\frac{e^2}{\hbar c} \approx \frac{1}{137}$, $v \sim 10^{-2}c \Rightarrow \frac{e^2}{\hbar v} \sim 1$

В.-Ф.: $\varkappa = \frac{\pi^2}{3e^2}\sigma T \sim \frac{\mu^2}{\hbar^2 v}\frac{\mu}{T}$

τ : τ_p — импульсная релаксация

τ_ε — энергетич.



$\tau = \tau_p$

$\cancel{\tau_\varepsilon = \infty}$

e-e: $\begin{cases} \varepsilon \sim T \\ \Delta\varepsilon \sim T \end{cases}$

$\tau_p \sim \tau_\varepsilon$

$\begin{cases} p \sim p_0 \\ \Delta p \sim p_0 \end{cases}$

τ — время жизни кв.-ч.

$$\Delta E \sim \frac{\hbar}{\tau} \ll |\xi| \sim T$$

$$\frac{\hbar}{\frac{\hbar\mu}{T^2}} \ll T \Rightarrow \boxed{T \ll \mu}$$

$$T \ll T_0 \sim 10^4 - 10^5 \text{ K}$$

V. e-ph рассеяние

$\vec{P}(\vec{r})$ – поляризация

$$\rho_{пол} = -\operatorname{div}\vec{P}$$

$$\operatorname{div}\vec{E} = 4\pi\rho = 4\pi\rho_{св} + 4\pi\underbrace{\rho_{пол}}_{-\operatorname{div}\vec{P}} \rightarrow \operatorname{div}\overbrace{(\vec{E}+4\pi\vec{P})}^{\vec{D}} = 4\pi\rho_{св}$$

$$V(\vec{r}) = -e\int Q(\vec{r}-\vec{r}\,')\operatorname{div}\vec{P}(\vec{r}\,')\,d^3\vec{r}\,'$$

Без экранирования: $Q = \dfrac{1}{|\vec{r}-\vec{r}\,'|}$

С учетом экр.: $Q(\vec{r}-\vec{r}\,') \approx a^2\delta(\vec{r}-\vec{r}\,')$

$$k = \frac{\omega}{c_s} \qquad a \sim 10^{-8}\,см \sim \frac{\hbar}{p_0}$$

$$V(\vec{r}) \sim \overbrace{-ea^2}^{V_{\vec{k}}}\;\overbrace{\operatorname{div}\vec{P}(\vec{r})}^{i\vec{k}}, \qquad \vec{P} \sim ne\vec{u}$$

$$V_{\vec{k}} \sim -ie^2a^2n\,\frac{\omega}{c_s}\,u_{\vec{k}}\,, \qquad u_{\vec{k}} \to a_{\vec{k}},\, a^+_{\vec{k}}$$

Испускание фонона: $\langle n_k+1|a_k^+|n_k\rangle = \sqrt{n_k+1}$

$$V_{\vec{p}-\hbar\vec{k},\vec{p}} \sim -ie^2a^2n\left(\frac{\hbar(n_k+1)}{NM\omega}\right)^{1/2}\frac{\omega}{c_s} \sim -ina^3\frac{e^2}{a}\left(\frac{\hbar\omega(n_k+1)}{N}\right)^{1/2}\cdot\frac{1}{c_s\sqrt{M}}$$

$$na^3\sim 1\,,\quad \frac{e^2}{a}\sim\frac{p_0^2}{m}\,,\quad \boxed{c_s\sqrt{M}\sim v\sqrt{m}}$$

e: a, m $\qquad p_0\sim\frac{\hbar}{a}\,,\quad v\sim\frac{p_0}{m}\sim\frac{\hbar}{ma}$

ph: M $\qquad \rho\propto M \qquad c_s\propto\rho^{-1/2}\propto M^{-1/2}$

$$\frac{c_s}{v}\sim\left(\frac{m}{M}\right)^{1/2}$$

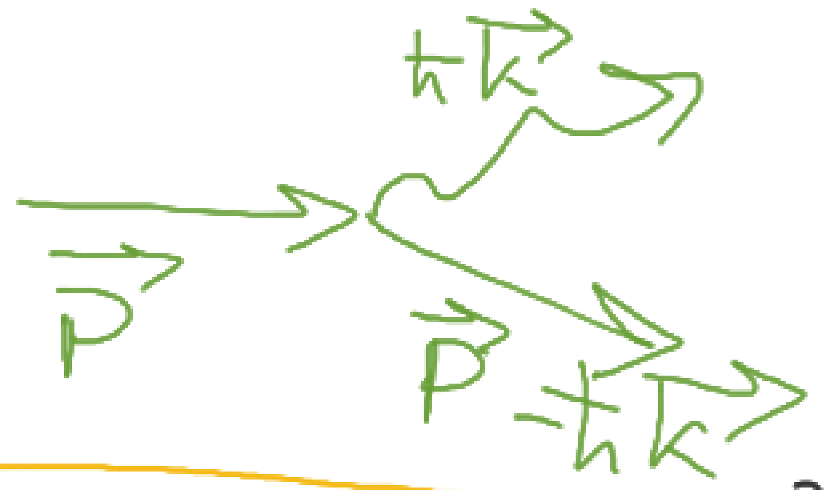
$$k_D\sim k_0$$

$$\frac{\hbar\omega_D}{\mu}\sim\frac{c_s}{v}\sim\left(\frac{m}{M}\right)^{1/2}\ll 1$$

$$\hbar\omega_D\sim c_s p_D$$

$$\mu\sim v p_0$$

$$V_{\vec p-\hbar\vec k,\vec p} \sim -iP_0\left(\frac{\hbar\omega(n_k+1)}{Nm}\right)^{1/2}$$



1) $T \gg \hbar\omega_D$

✓

$$w=\frac{2\pi}{\hbar}\int |V_{\vec p-\hbar\vec k,\vec p}|^2\,\delta\big(\varepsilon(\vec p)-\varepsilon(\vec p-\hbar\vec k)-\hbar\omega(\vec k)\big)\,\frac{V\,d^3k}{(2\pi)^3}$$

$$\delta\left(\frac{p^2}{2m}-\frac{(\vec p-\hbar\vec k)^2}{2m}-\hbar\omega(\vec k)\right)=\delta\left(\frac{\hbar\vec p\vec k}{m}-\frac{\hbar^2k^2}{2m}-\hbar\omega(\vec k)\right)$$

$$=\frac{m}{\hbar pk}\,\delta\left(\cos\varphi-\frac{\hbar k}{2p}-\cancel{\frac{m\omega}{pk}}\right)$$

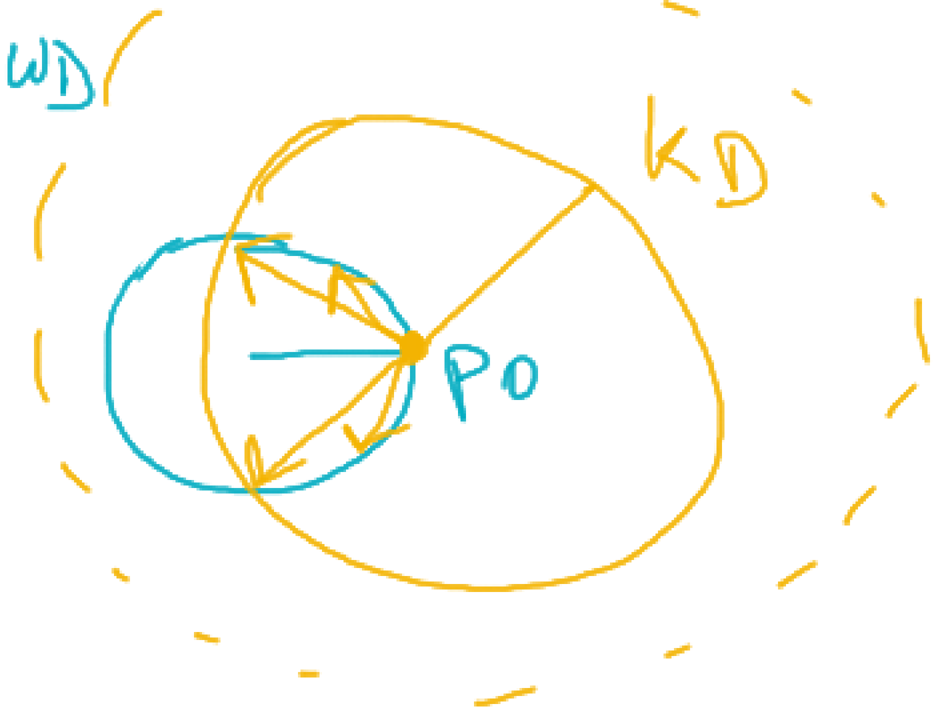
$\frac{\hbar k}{p_0}\ll 1$ $\quad\cos\varphi\ll 1$

$$\frac{\hbar k}{p}\sim\frac{\hbar k_D}{p_0}\sim 1,\qquad \frac{m\omega}{pk}\sim\frac{mc_s}{p_0}\sim\frac{c_s}{v}\ll 1$$

$$\boxed{\cos\varphi\sim 1}$$

$$\delta(\ldots)\,d^3k\sim\frac{m}{\hbar p k_D}\,k_D^3\sim\frac{mk_D^2}{\hbar p_0}\sim\frac{mp_0}{\hbar^3}$$

$T \gg \hbar\omega_D$



"дебай - сфера"

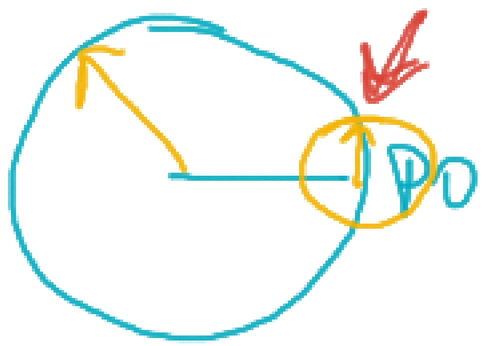
$$\underbrace{n_k+1} \approx n_k \approx \frac{T}{\hbar\omega_D}$$

$$n_k = \frac{1}{e^{\frac{\hbar\omega(k)}{T}} - 1} \approx \underbrace{\frac{T}{\hbar\omega_D}} \gg 1$$

$$\left[\ \underbrace{\frac{1}{\tau}} \sim W \sim \frac{1}{\hbar} p_0^2 \frac{\cancel{\hbar\omega_D}\,\frac{T}{\cancel{\hbar\omega_D}}}{N \cancel{m}} V \frac{\cancel{m} p_0}{\hbar^3} \sim\right.$$

$$\sim \frac{\cancel{p_0^3}\, T}{\hbar^4 \cancel{n}} \sim \frac{T}{\hbar}$$

$T \ll \hbar\omega_D$:



$n_k \sim 1$

$$\sigma \sim \frac{n_e e^2 \tau}{m} \sim \frac{\mu}{\hbar}\frac{\mu}{T} \quad \Big| \quad \rho \propto T$$

$$\underbrace{\hbar\omega_D} \ll T$$

$$\frac{p^4}{\hbar^2 m^2 v} \sim \frac{p^3}{\hbar m}$$

$$\underbrace{\varkappa} \approx \frac{\pi^2}{3e^2}\underbrace{\sigma} T \sim \boxed{\frac{\mu^2}{\hbar^2 v}} = const$$

2. $T \ll \hbar\omega_D$

$\hbar\omega \sim T$

$\tau_\varepsilon \sim \tau$

p_0

$$\hbar k \sim \frac{\hbar\omega}{c_s} \sim \frac{T}{c_s}$$

$$\frac{\hbar k}{p_0} \sim \frac{T}{c_s p_0} \sim \frac{T}{\hbar\omega_D} \ll 1$$

$\tau_p \gg \tau$

~~Закон В.-Ф.~~

Разные τ для σ и $\varkappa$.

$$\sigma \to \tau_e, \qquad \varkappa \to \tau_t$$

$$\tau_t \sim \frac{1}{W}, \qquad W \sim \left(\frac{T}{\hbar\omega_D}\right)^2 \frac{T}{\hbar}$$

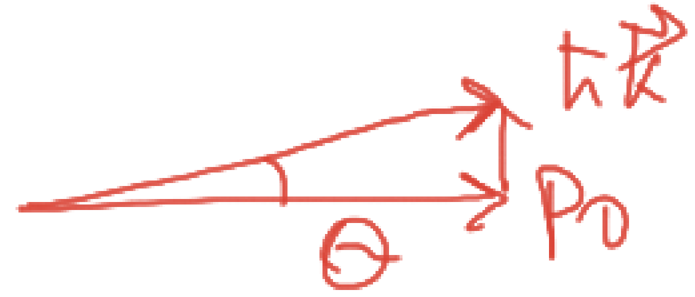
$$\varkappa \sim C_e \underbrace{l v}_{v^2\tau_t} \sim \frac{m p_0}{\hbar^3} T \left(\frac{p_0}{m}\right)^2 \left(\frac{\hbar\omega_D}{T}\right)^2 \frac{\hbar}{T} \sim \frac{p_0^3}{\hbar^2 m}\left(\frac{\hbar\omega_D}{T}\right)^2 \sim$$

$$\sim \varkappa\,(T \gg \hbar\omega_D)\left(\frac{\hbar\omega_D}{T}\right)^2$$

σ, τ_e

$$\frac{1}{\tau} = \int W(\theta)\underbrace{(1-\cos\theta)}\frac{d\Omega}{4\pi}$$

$$\underbrace{\theta \sim \frac{\hbar k}{p_0} \sim \frac{T}{\hbar\omega_D}} \ll 1$$



$$\frac{1}{\tau_e} \sim \overline{W(\theta)\theta^2} \sim \boxed{\left(\frac{T}{\hbar\omega_D}\right)^4} \frac{T}{\hbar}$$

$$\sigma \sim \underbrace{\sigma(T \gg \hbar\omega_D)}_{\propto \frac{1}{T}} \underbrace{\left(\frac{\hbar\omega_D}{T}\right)^4}, \quad \rho \propto T^5$$

(закон Блоха-Грюнайзена)

$$\frac{\varkappa}{\sigma} \sim \frac{T}{e^2}\overbrace{\left(\frac{T}{\hbar\omega_D}\right)^2} \leftarrow$$